Beam loading effects in collective beam instabilities

S.Y. Lee
Indiana University and GSI

1. Comments on the beam loading effects in collective beam instabilities
2. Unified approach on collective beam instability and nonlinear beam dynamics

1. Example: Collective beam Instabilities

When particle motion is governed by a Hamiltonian system, that includes field generated by the beam, the distribution function obeys the Vlasov equation (Liouville theorem) :

$$\frac{d\Psi}{dt} = \frac{\partial \Psi}{\partial t} + \dot{\theta}\frac{\partial \Psi}{\partial \theta} + \dot{\delta}\frac{\partial \Psi}{\partial \delta} = 0.$$

We consider a coasting beam with perturbation:

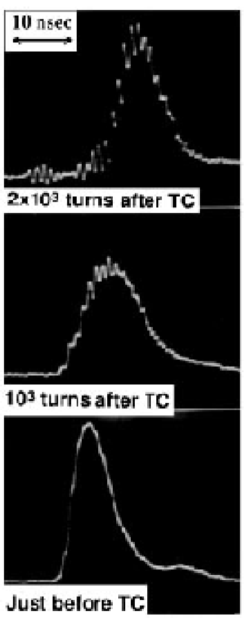
$$\Psi(\theta, \delta; t) = \Psi_0(\delta) + \Delta\Psi(\delta) e^{i(n\theta - \Omega t)},$$

$$\Delta I_n = \frac{eqN_{\rm B}\omega_0}{2\pi}\int \Delta\Psi_n d\delta e^{i(n\theta-\Omega t)} = I_0 \int \Delta\Psi_n d\delta e^{i(n\theta-\Omega t)},$$

In the presence of longitudinal impedance, the induced voltage is

$$\Delta V_n = \Delta I_n Z_0^{\|}(\Omega)$$

$$\dot{\delta} = -\frac{e\omega Z_0^{\|}(\Omega)\Delta I_n}{2\pi\beta^2 E} = -\frac{e\omega Z_0^{\|} I_0}{2\pi\beta^2 E}\int \Delta\Psi_n d\delta e^{i(n\theta-\Omega t)},$$



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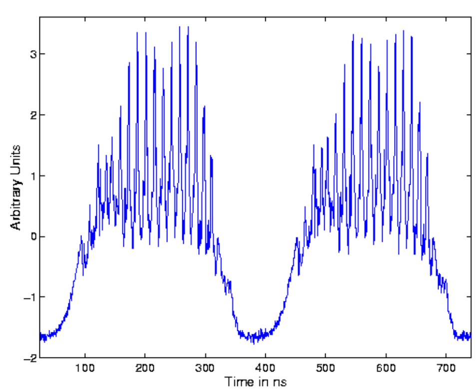


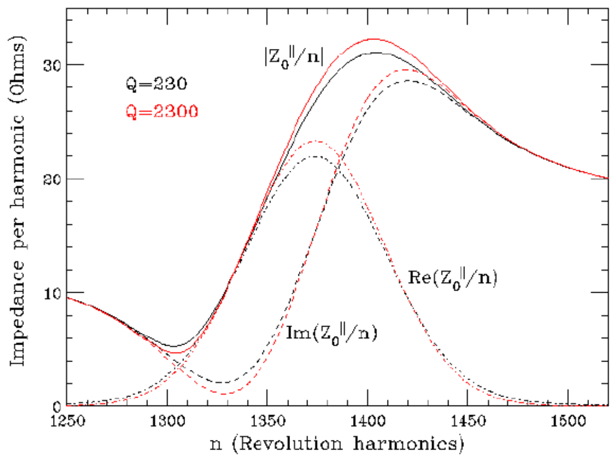
Characteristic Properties of KEK PS impedance sources

	f_r (GHz)	Q	$R_{\rm sh}$ (Ω)	$R_{\rm sh}/Q$ (Ω)
BPM	0.636/0.667	77/2650	$1.5\times10^3/2.6\times10^4$	19.4/9.8
	/1.13	/3769	$/6.2\times10^4$	/16.3
	1.498/1.377	230/8222	$5.3\times10^3/3.3\times10^3$	23/40
CVC	/1.44	/4868	$/1.39\times10^5$	/28.8
	/1.84	/4078	$/1.94\times10^5$	/46.6

$$Z(n)_{\rm eff} = \int dn_r \frac{e^{-(n_r-n_{r0})^2/(2\sigma_n^2)}}{\sqrt{2\pi}\sigma_n}\frac{MR_{\rm sh}}{1+iQ\left(\frac{n_r}{n}-\frac{n}{n_r}\right)}$$

$$= \frac{\sqrt{\pi}MR_{\rm sh}}{2Q\sqrt{1-1/(4Q^2)}}\left[n'_+ w(n'_+ - n'_{r0}) - n'_- w(n'_- - n'_{r0})\right],$$

$$n'_\pm = \frac{n}{\sqrt{2}\sigma_n}\left(\frac{i}{2Q} \pm \sqrt{1-\frac{1}{4Q^2}}\right) \quad \text{and} \quad n'_{r0} = \frac{n_{r0}}{\sqrt{2}\sigma_n},$$



$$\tau_{\rm transit} = \frac{\sin \frac{n\omega_0 \ell}{2\beta c}}{\frac{n\omega_0 \ell}{2\beta c}},$$

Impedance per harmonic of the KEK PS, including the space-charge impedance and the impedance of the BPMs and CVCs at 56 quadrupoles. The resonances of the 56 BPMS and CVCs are assumed to have a random rms spread in center frequency of σ_n, at 2.5% of the center harmonic n_{r0}=1370. The resulting effective Q-value is n_{r0}/σ_{r0}=40.

$$\Delta V_n = \Delta I_n Z_0^{\|}(\Omega)$$

The induced voltage is current times the impedance. However, this formula is valid only when the steady state is reached! We examine the requirement of a steadv state!

$$Z = \left(\frac{1}{R_{\rm sh}} + j\omega C_{\rm eq} + \frac{1}{j\omega L_{\rm eq}}\right)^{-1} = \frac{R_{\rm sh}}{1 + j{\rm Q}(\omega/\omega_{\rm r} - \omega_{\rm r}/\omega)} \approx R_{\rm sh}\cos\psi e^{-j\psi},$$

$$V_{\rm b} = \frac{1}{2}V_{\rm b0} + V_{\rm b0}(e^{-(\lambda+j\phi)} + e^{-2(\lambda+\phi)} + \cdots) = V_{\rm b0}(-\frac{1}{2} + \frac{1}{1-e^{-(\lambda+j\phi)}}),$$

$$\lambda = T_{\rm b}/T_{\rm f} = \omega_{\rm r}T_{\rm b}/2{\rm Q_L} \ll 1$$

$$\phi = (\omega - \omega_{\rm r})T_{\rm b} = +(T_{\rm b}/T_{\rm f})\tan\psi = +\lambda\tan\psi.$$

$$V_{\rm b} = I_{\rm i}R_{\rm sh}\lambda(-\frac{1}{2} + \frac{1}{1-e^{-(\lambda+j\phi)}}) \approx I_{\rm i}\frac{R_{\rm sh}}{(1+d)}\cos\psi e^{-j\psi} \quad (\lambda \to 0),$$

We consider the mode number *n*, i.e. the accelerator is divided into bins with the time slice of $T_b=T_0/n$. The filling time of a cavity is $T_f=2Q/\omega_r$. Thus

1) For induced voltage within the bunch, we find $\lambda=\pi/Q$. The number of slices needed to achieve steady state is about $\ln(2)/\lambda=Q/5$. For example, the microwave instability at the KEK PS needs at least 46 slices (Q=230) to reach the steady state! However, the beam is composed only about 20 slices for the instability at 1.23 GHz!
2) For the induced voltage in the second revolution, we find $\lambda=n\pi/Q$. Thus the wake field is most-likely decayed in the second revolution. Thus the instability is a single bunch effect.

It would be very helpful to carry out controlled experiments with known impedance, and observe instability from a primary bunch and a witness bunch to determine the effect of effective impedances.

2. Unified approach:

$$\frac{d\Psi}{dt} = \frac{\partial\Psi}{\partial t} + \dot{\theta}\frac{\partial\Psi}{\partial\theta} + \dot{\delta}\frac{\partial\Psi}{\partial\delta} = 0.$$

$$\dot{\delta} = -\frac{e\omega Z_0^{\|}(\Omega)\Delta I_n}{2\pi\beta^2 E} = -\frac{e\omega Z_0^{\|} I_0}{2\pi\beta^2 E}\int \Delta\Psi_n d\delta e^{i(n\theta-\Omega t)},$$

$$H = H_0(\theta,\delta) - i\frac{e\omega Z_0^{\|}/n}{2\pi\beta^2 E}\left(\int I_0 \Delta\Psi_n d\delta\right)e^{i(n\theta-\Omega t)}$$

The perturbation is most important when the time modulation frequency produces a resonance of the dynamical system. In this case, one can solve the Hamiltonian in the resonance rotating frame and carry out equilibrium distribution function in the resonance rotating frame. Unfortunately, there are difficulties: (1) The induced current depends on the resonance, i.e. one needs a self-consistent bootstrap; (2) The calculation is much more complicated.

I still have not found any breakthrough since 1999.