

Resistive–Wall Impedance for

Arbitrary Beam Energies

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OUTLINE

- **Theoretical Model**
- **Longitudinal Coupling Impedance**
Arbitrary wall thickness
Beam Shielding via a thin conducting wall
- **Transverse Coupling Impedance**
Thick beam pipe wall
- **Transverse Coupling Impedance**
Arbitrary wall thickness
Beam Shielding via a thin conducting wall
- **Conclusions**

Wave Equations

- Maxwell's Equations

$$\nabla^2 \vec{\mathbf{B}} - \mu\epsilon \frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2} - \mu\mathbf{S} \frac{\partial \vec{\mathbf{B}}}{\partial t} = -\mu \vec{\nabla} \times \vec{\mathbf{j}}_i$$
$$\nabla^2 \vec{\mathbf{E}} - \mu\epsilon \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} - \mu\mathbf{S} \frac{\partial \vec{\mathbf{E}}}{\partial t} = \mu \frac{\partial \vec{\mathbf{j}}_i}{\partial t} + \frac{\vec{\nabla} \rho_i}{\epsilon}$$

- Boundary Conditions

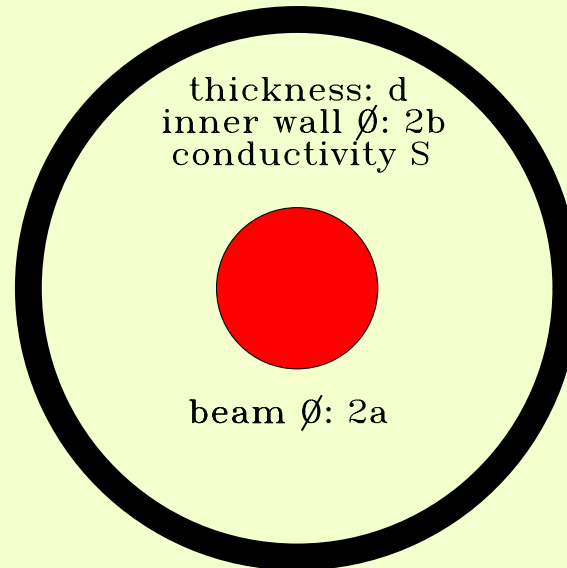
- Thick Wall such that $d \gg \delta_s$: Impedance B. C.

$$\vec{\mathbf{E}}_t = \mathbf{Z}_m \vec{\mathbf{j}}_s = \mathbf{Z}_m \hat{\mathbf{n}} \times \vec{\mathbf{H}}_t \quad \mathbf{Z}_m = \frac{1 - i}{\mathbf{S} \delta_s}, \quad \delta_s = \sqrt{\frac{2}{\mu \omega \mathbf{S}}}$$

Impedance or Leontovich B. C. is not valid down to zero frequency

- Arbitrary Wall Thickness: ($d \leq \delta_s$): Max. field eqns.

Beam-Pipe Structures



$$\rho(\mathbf{r}, \mathbf{t}) = \sigma(\mathbf{r}, \theta) \delta(\mathbf{z} - \beta c \mathbf{t}) = \frac{Q^2}{\pi a^2} \mathbf{H} \left(\mathbf{a} - |\vec{\mathbf{r}} - \vec{\mathbf{d}}| \right) \delta(\mathbf{z} - \mathbf{v} \mathbf{t})$$

$$\sigma_0(\mathbf{r}, \theta) = \frac{Q^2}{\pi a^2} \quad \sigma_1(\mathbf{r}, \theta) = \frac{P}{\pi a^2} \cos(\theta - \theta_0) \delta(\mathbf{a} - \mathbf{r})$$

Machine Parameters

	SIS18	SIS100	HESR
Circumference C	216 m	1080 m	570 m
Conductivity S in $(\Omega \text{ m})^{-1}$	10^6	10^6	10^6
Pipe Radius b	10 cm	1 cm	5 cm
Wall Width in the Dipoles d_w	0.3 mm	0.3 mm	1 cm
Reference Energy γ_0	1.0122	1.098	2
Reference Energy β_0	0.155	0.413	0.866
Rev. Frequency f_0 in kHz	214	113	453

Longitudinal Impedance: Arbitrary d

$$\mathbf{Z}_{\parallel,0}^{(\text{rw})}(\omega) = (1 - i) \frac{nZ_0\beta\delta_s^*}{2\sqrt{n}b} \frac{4I_1^2(\sigma_0 a)}{\sigma_0^2 a^2 I_0^2(\sigma_0 b)} \times$$

$$\times \frac{\mathbf{1} + \eta \frac{\mathbf{K}_1(\sigma_0 h)}{\mathbf{K}_0(\sigma_0 h)} \tanh \underline{\sigma} \mathbf{d}}{\tanh \underline{\sigma} \mathbf{d} + \eta \mathbf{V}_1 + \eta^2 \mathbf{V}_2 \tanh \underline{\sigma} \mathbf{d}}$$

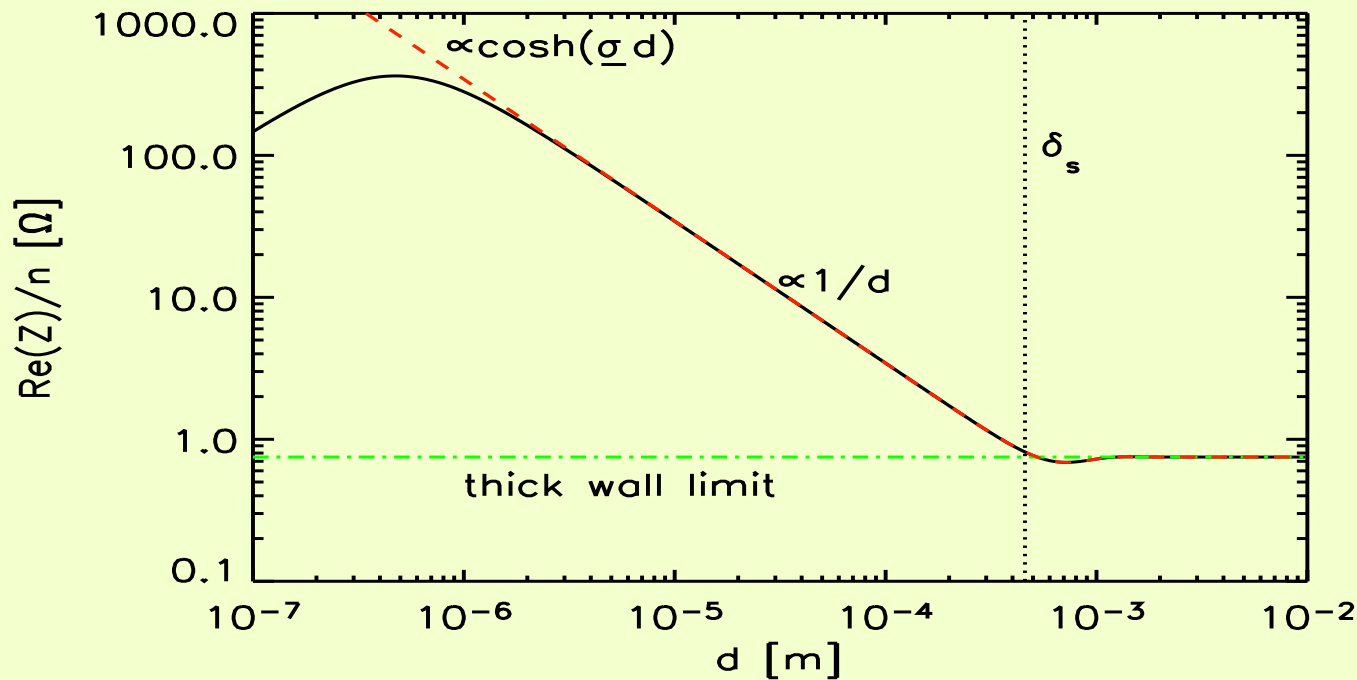
$$\mathbf{V}_1 = \frac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)} + \frac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}, \quad \mathbf{V}_2 = \frac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)} \frac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}$$

$$\sigma_0 = \frac{\mathbf{k}_z}{\gamma_0}, \quad \underline{\sigma} = \frac{\mathbf{k}_z}{\underline{\gamma}}, \quad \underline{\gamma}^{-2} = \gamma_0^{-2} - \mathbf{i} \frac{\mu_0 \mathbf{S} \omega}{\mathbf{k}_z^2}, \quad \eta = \frac{\omega \epsilon_0 \gamma_0}{\mathbf{i} \underline{\gamma} (\mathbf{S} - \mathbf{i} \omega \epsilon_0)}$$

Ahmed Al-Khateeb et al, Phys. Rev. E (2005)

Longitudinal Resistive Wall Impedance $Z^{(rw)}(\omega)$

SIS18 for $n = 1$, $\gamma_0 = 2$: $C = 216$ m, $a = 1$ cm, $b = 10$ cm, $d_w = 0.3$ mm:
 $Z^{(rw)}/n$ remains below 10Ω



$$Z^{(rw)}_{\parallel,0}(\omega) \approx (1 - i) \frac{nZ_0\beta\delta_s^*}{2\sqrt{nb}} \frac{4I_1^2(\sigma_0 a)}{\sigma_0^2 a^2 I_0^2(\sigma_0 b)} \coth \underline{\sigma} d \quad [\text{Any } d, \delta_s k_z \ll 1]$$

Transmission coefficient

Beam shielding via metallic walls: : **Transmission coefficient** τ

- Ahmed Al-Khateeb et al, Phys. Rev. E (2005)

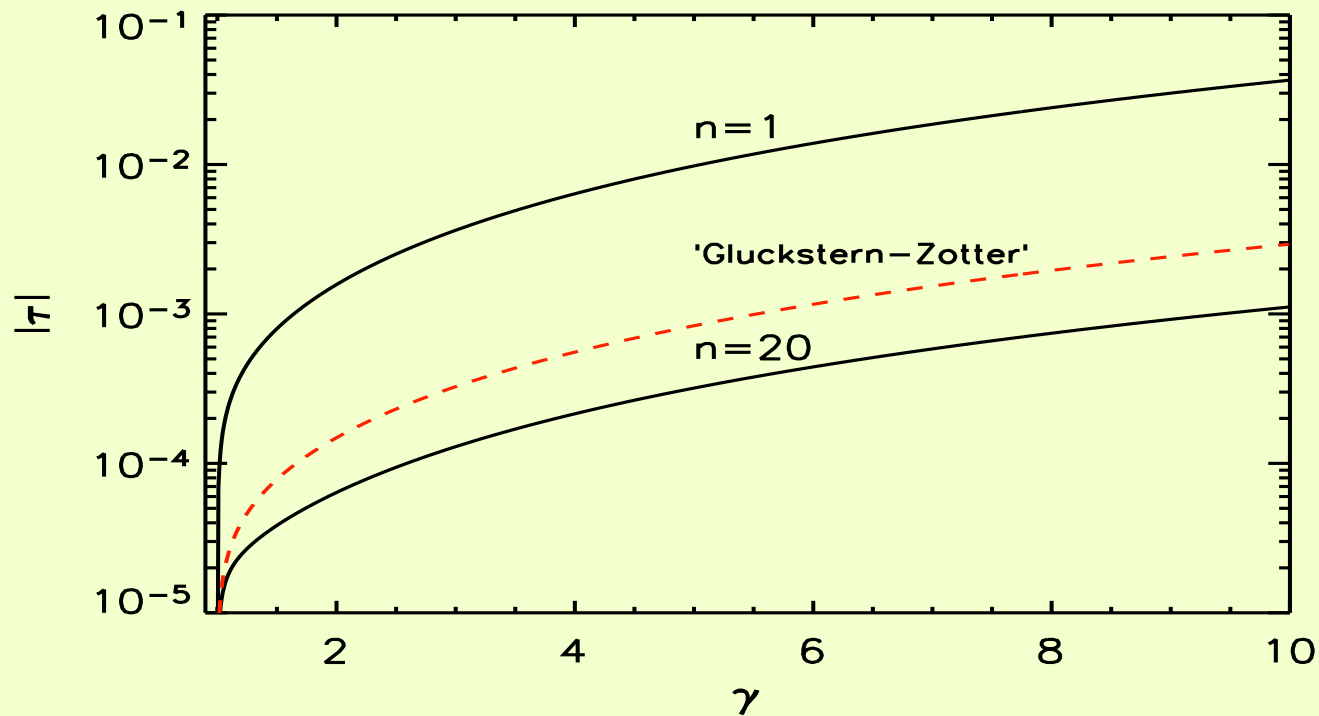
$$\tau^{-1} = \sqrt{\frac{h}{b}} \sigma_0 b \mathbf{K}_1(\sigma_0 h) \mathbf{I}_0(\sigma_0 b) \sinh \underline{\sigma} d \left[\coth \underline{\sigma} d + \frac{\mathbf{K}_0(\sigma_0 h)}{\eta \mathbf{K}_1(\sigma_0 h)} + \frac{\mathbf{I}_1(\sigma_0 b)}{\mathbf{I}_0(\sigma_0 b)} \left(\eta + \frac{\mathbf{K}_0(\sigma_0 h)}{\mathbf{K}_1(\sigma_0 h)} \coth \underline{\sigma} d \right) \right]$$

- Gluckstern and Zotter, Phys. Rev. STAB (2001)

$$\tau^{-1} \approx 1 + \frac{\beta^2 \mathbf{k}_z^2 b}{2} d + i \frac{2bd}{\beta^2 \gamma_0^2 \delta_s^2} \ln \frac{\mathbf{k}_z b}{\gamma_0}$$

Transmission coefficient: SIS18

$C = 216$ m, $a = 1$ cm, $b = 10$ cm, $d_w = 0.3$ mm



$n=1$ or $\omega = \omega_0$: Up to $\gamma_0 = 6$, τ reaches 1%

Thick Wall Transverse Impedance

$$\mathbf{Z}_{1,\perp}^{(sc+rw)} = \frac{i\mathbf{Z}_0 L \mathbf{I}_1^2(\sigma_0 a)}{\pi a^2 \gamma_0^2 \beta} \left[\frac{\mathbf{K}_1(\sigma_0 a)}{\mathbf{I}_1(\sigma_0 a)} - \frac{\mathbf{K}_1(\sigma_0 b) + \frac{i\gamma_0 \beta \mathbf{Z}_m}{\mathbf{Z}_0} \mathbf{K}'_1(\sigma_0 b)}{\mathbf{F}} \right. \\ \left. \frac{\frac{i\mathbf{Z}_m \gamma_0^3}{\mathbf{F} \mathbf{Z}_0 \beta k_z^2 b^2} \frac{\mathbf{I}_1(\sigma_0 b) \mathbf{G}}{\mathbf{I}'_1(\sigma_0 b) - \frac{i\mathbf{Z}_m}{\mathbf{Z}_0 \beta \gamma_0} \mathbf{I}_1(\sigma_0 b)}}{\mathbf{F}} \right].$$

$$\mathbf{F} = \mathbf{I}_1(\sigma_0 b) + i \frac{\gamma_0 \beta \mathbf{Z}_m}{\mathbf{Z}_0} \mathbf{I}'_1(\sigma_0 b) - i \frac{\mathbf{Z}_m \gamma_0^3}{\mathbf{Z}_0 \beta k_z^2 b^2} \frac{\mathbf{I}_1^2(\sigma_0 b)}{\mathbf{I}'_1(\sigma_0 b) - i \frac{\mathbf{Z}_m}{\mathbf{Z}_0 \beta \gamma_0} \mathbf{I}_1(\sigma_0 b)}$$

$$\mathbf{G} = \mathbf{K}_1(\sigma_0 b) + \frac{\beta^2 k_z^2 a b}{\gamma_0^2 \mathbf{I}_1(\sigma_0 a)} \left[\mathbf{I}'_1(\sigma_0 a) \left(\mathbf{K}'_1(\sigma_0 b) - i \frac{\mathbf{Z}_m \mathbf{K}_1(\sigma_0 b)}{\mathbf{Z}_0 \beta \gamma_0} \right) - \right. \\ \left. \frac{\mathbf{K}_1(\sigma_0 b)}{\mathbf{I}_1(\sigma_0 b)} \left(\mathbf{I}'_1(\sigma_0 b) - i \frac{\mathbf{Z}_m \mathbf{I}_1(\sigma_0 b)}{\mathbf{Z}_0 \beta \gamma_0} \right) \right].$$

TM $\mathbf{Z}_{\perp,1}^{(\text{rw})}(\omega)$ for Thick Wall

$Z_m = 0$, TM + TE \rightarrow TM

$$\mathbf{Z}_{1,\perp,\text{TM+TE}}^{(\text{sc})} \rightarrow \mathbf{Z}_{1,\perp,\text{TM}}^{(\text{sc})} = \mathbf{i} \frac{\mathbf{Z}_0 \mathbf{L} \mathbf{I}_1^2(\sigma_0 \mathbf{a})}{\pi \mathbf{a}^2 \gamma_0^2 \beta} \left[\frac{\mathbf{K}_1(\sigma_0 \mathbf{a})}{\mathbf{I}_1(\sigma_0 \mathbf{a})} - \frac{\mathbf{K}_1(\sigma_0 \mathbf{b})}{\mathbf{I}_1(\sigma_0 \mathbf{b})} \right]$$

$$\mathbf{Z}_{1,\perp,\text{TM+TE}}^{(\text{sc})}(\omega) = \mathbf{Z}_{1,\perp,\text{TM}}^{(\text{sc})}(\omega) = \mathbf{i} \frac{\mathbf{Z}_0 \mathbf{L}}{2\pi\beta\gamma_0^2} \left[\frac{1}{\mathbf{a}^2} - \frac{1}{\mathbf{b}^2} \right]$$

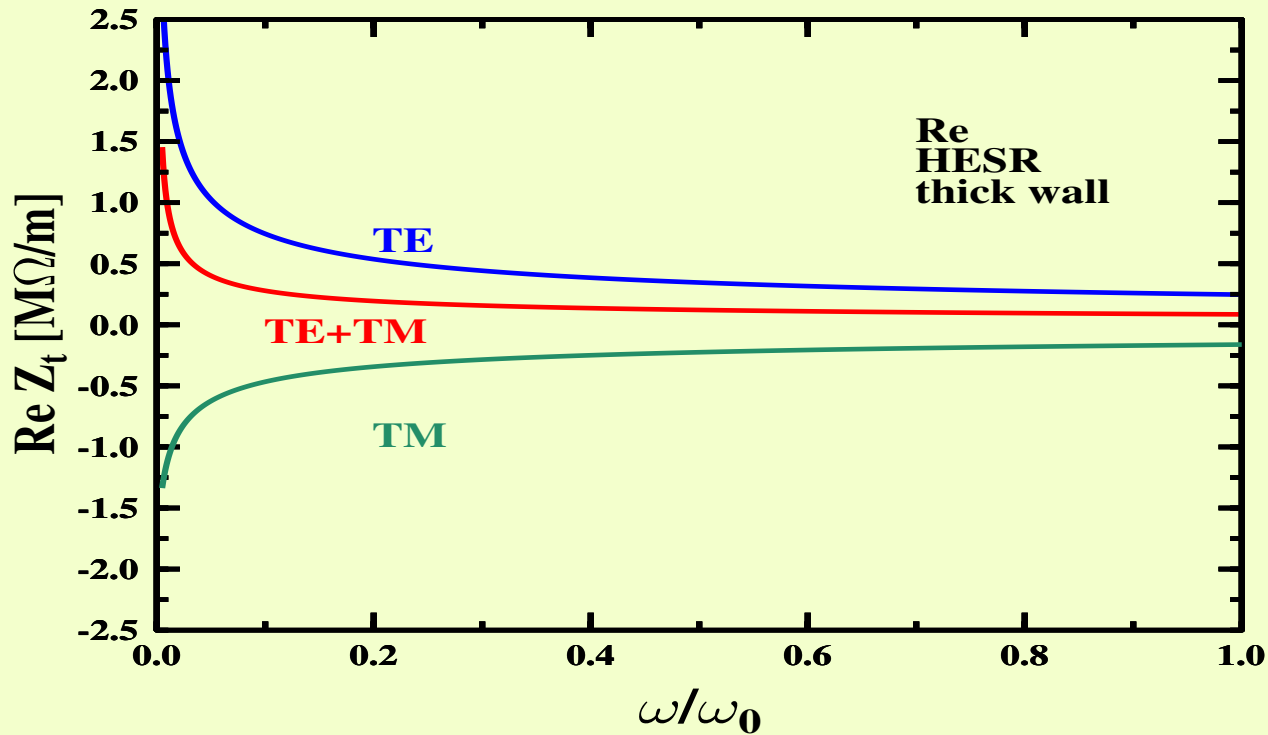
$Z_m \neq 0$, TM only

$$\mathbf{Z}_{1,\perp,\text{TM}}^{(\text{rw})}(\omega) = \mathbf{i} \frac{\mathbf{Z}_0 \mathbf{L} \mathbf{I}_1^2(\sigma_0 \mathbf{a})}{\pi \mathbf{a}^2 \gamma^2 \beta} \left[\frac{\mathbf{K}_1(\sigma_0 \mathbf{b})}{\mathbf{I}_1(\sigma_0 \mathbf{b})} - \frac{\mathbf{K}_1(\sigma_0 \mathbf{b}) + \mathbf{i} \frac{\gamma\beta \mathbf{Z}_m}{\mathbf{Z}_0} \mathbf{K}'_1(\sigma_0 \mathbf{b})}{\mathbf{I}_1(\sigma_0 \mathbf{b}) + \mathbf{i} \frac{\gamma\beta \mathbf{Z}_m}{\mathbf{Z}_0} \mathbf{I}'_1(\sigma_0 \mathbf{b})} \right]$$

$$\text{Re} \left(\mathbf{Z}_{1,\perp,\text{TM}}^{(\text{rw})}(\omega) \right) \approx -\frac{2}{\mathbf{b}^2 \mathbf{k}} \frac{\mathbf{L} \mathbf{Z}_0 \omega \delta_s}{4\pi \mathbf{b} \mathbf{c}} = -\frac{2}{\mathbf{b}^2 \mathbf{k}} \text{Re} \left(\mathbf{Z}_{0,\parallel,\text{TM}}^{\text{rw}}(\omega, \gamma_0 \rightarrow \infty) \right)$$

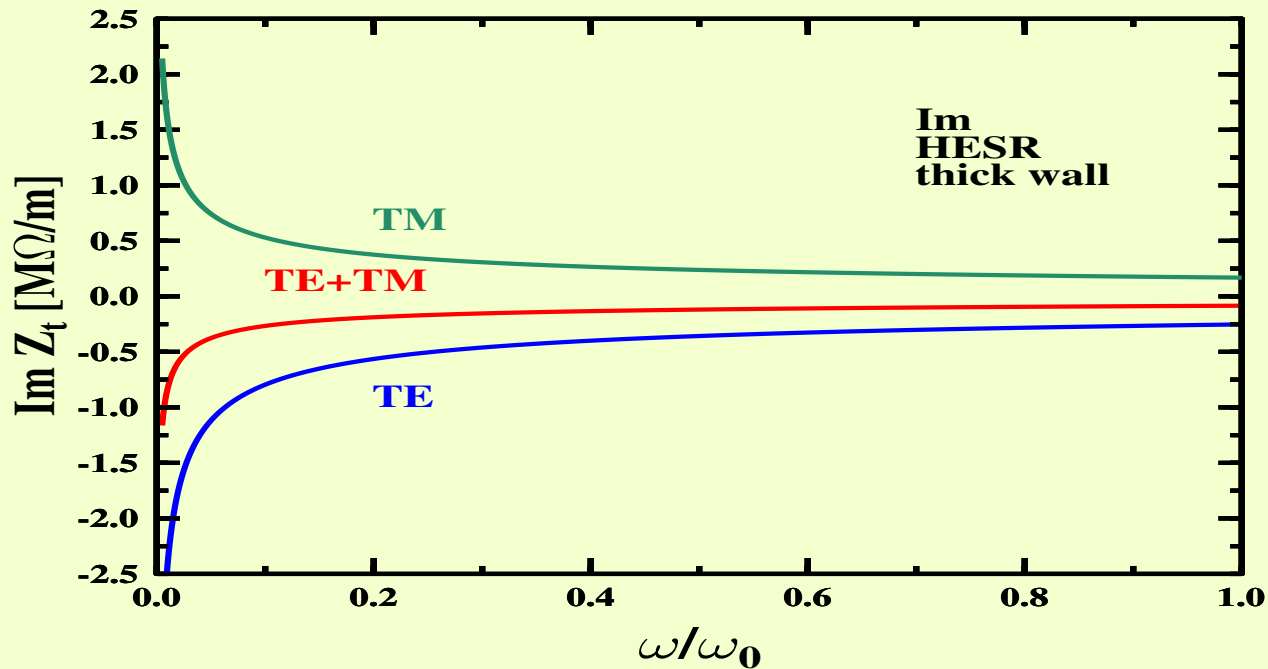
Thick Wall: Real Part of $Z_{\perp,1}^{(rw)}(\omega)$ for HESR

$C = 570$ m, $b = 5$ cm, $a = 1$ cm, $\gamma_0 = 2$



Thick Wall: Imaginary Part of $Z_{\perp,1}^{(rw)}(\omega)$ for HESR

$C = 570 \text{ m}$, $b = 5 \text{ cm}$, $a = 1 \text{ cm}$, $\gamma_0 = 2$



Note: $-i \rightarrow j$

Transverse Impedance: Arbitrary Wall Thickness

$$\mathbf{Z}_{1,\parallel}(\omega) = i \frac{n\mathbf{Z}_0}{2\beta\gamma_0^2} \frac{4I_1^2(\sigma_0 a)}{a^2} \mathbf{F}, \quad \mathbf{Z}_{1,\perp}(\omega) = \frac{\mathbf{Z}_{1,\parallel}(\omega)}{\mathbf{k}_z}$$

$$\mathbf{f}_0 = \frac{P\sigma_0 a K_1(\sigma_0 a)}{\epsilon_0 \gamma_0 \beta c \pi a^2}, \quad \mathbf{a}_{00} = \frac{\omega}{k_z \underline{\sigma} h} \left(1 - \frac{\eta_3 \underline{\sigma}}{\sigma_0} \right), \quad \mathbf{h} = \mathbf{b} + \mathbf{d}$$

$$\mathbf{f}_3 = \left(1 - \frac{\underline{\gamma}^2}{\gamma_0^2} \right) \left(\frac{K_1(\underline{\sigma} h)}{I_1(\underline{\sigma} h)} - \frac{K_1(\underline{\sigma} b)}{I_1(\underline{\sigma} b)} \right), \quad \eta_3 = \frac{\omega \epsilon_0 \gamma_0}{i \underline{\gamma} (S_w - i \omega \epsilon_w)}$$

$$\mathbf{F} = \frac{I_1(\underline{\sigma} b)}{I_1(\sigma_0 b)} \frac{\alpha_{22}\beta_{11} - \alpha_{12}\beta_{22}}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}} + \frac{K_1(\underline{\sigma} b)}{I_1(\sigma_0 b)} \frac{\alpha_{11}\beta_{22} - \alpha_{21}\beta_{11}}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}} + \frac{K_1(\sigma_0 a)}{I_1(\sigma_0 a)} - \frac{K_1(\sigma_0 b)}{I_1(\sigma_0 b)}$$

Transverse Impedance: Arbitrary Wall Thickness

$$\alpha_{11} = c_{11} \frac{I_1(\underline{\sigma}b)}{I_1(\sigma_0b)} + c_{21} - \eta_3 \frac{K'_1(\sigma_0h)}{K_1(\sigma_0h)} \frac{a_{11}}{a_{00}} \frac{I_1(\underline{\sigma}h)}{K_1(\sigma_0h)}$$

$$\alpha_{12} = c_{11} \frac{K_1(\underline{\sigma}b)}{I_1(\sigma_0b)} + c_{22} - \eta_3 \frac{K'_1(\sigma_0h)}{K_1(\sigma_0h)} \frac{a_{11}}{a_{00}} \frac{K_1(\underline{\sigma}h)}{K_1(\sigma_0h)}$$

$$\beta_{11} = c_{11} \frac{K_1(\sigma_0b)}{I_1(\sigma_0b)} - c_{12}, \quad \beta_{22} = e_{11} \frac{K_1(\sigma_0b)}{I_1(\sigma_0b)} - e_{33}$$

$$\alpha_{21} = e_{11} \frac{I_1(\underline{\sigma}b)}{I_1(\sigma_0b)} - d_{11} - d_{21} \frac{I_1(\underline{\sigma}h)}{K_1(\sigma_0h)},$$

$$\alpha_{22} = e_{11} \frac{K_1(\underline{\sigma}b)}{I_1(\sigma_0b)} - d_{12} - d_{21} \frac{K_1(\underline{\sigma}h)}{K_1(\sigma_0h)}$$

$$\mathbf{c}_{11} = f_3 I_1(\sigma_0b) - \eta_3 \frac{b}{h} \frac{I'_1(\sigma_0b)}{I_1(\sigma_0b)} \frac{a_{12}}{a_{00}}, \quad \mathbf{c}_{12} = f_3 K_1(\sigma_0b) - \eta_3 \frac{b}{h} \frac{K'_1(\sigma_0b)}{I_1(\sigma_0b)} \frac{a_{12}}{a_{00}}$$

$$\mathbf{c}_{21} = \frac{a_{11}}{a_{00}} \frac{I'_1(\underline{\sigma}h)}{K_1(\sigma_0h)} + \frac{a_{12}}{a_{00}} \frac{b}{h} \frac{I'_1(\underline{\sigma}b)}{I_1(\sigma_0b)}, \quad \mathbf{c}_{22} = \frac{a_{11}}{a_{00}} \frac{K'_1(\underline{\sigma}h)}{K_1(\sigma_0h)} + \frac{a_{12}}{a_{00}} \frac{b}{h} \frac{K'_1(\underline{\sigma}b)}{I_1(\sigma_0b)}$$

Transverse Impedance: Arbitrary Wall Thickness

$$\mathbf{d}_{11} = \frac{a_{21}}{a_{00}} \frac{I'_1(\underline{\sigma}h)}{K_1(\sigma_0 h)} + \frac{a_{22}}{a_{00}} \frac{b}{h} \frac{I'_1(\underline{\sigma}b)}{I_1(\sigma_0 b)}$$

$$\mathbf{d}_{12} = \frac{a_{21}}{a_{00}} \frac{K'_1(\underline{\sigma}h)}{K_1(\sigma_0 h)} + \frac{a_{22}}{a_{00}} \frac{b}{h} \frac{K'_1(\underline{\sigma}b)}{I_1(\sigma_0 b)}$$

$$\mathbf{e}_{11} = \frac{\eta_3 a_{22}}{a_{00}} \frac{b}{h} \frac{I'_1(\sigma_0 b)}{I_1(\sigma_0 b)}, \quad \mathbf{e}_{33} = \frac{\eta_3 a_{22}}{a_{00}} \frac{b}{h} \frac{K'_1(\sigma_0 b)}{I_1(\sigma_0 b)}$$

$$\mathbf{d}_{21} = f_3 K_1(\sigma_0 b) - \frac{\eta_3 K'_1(\sigma_0 h)}{K_1(\sigma_0 h)} \frac{a_{21}}{a_{00}}$$

$$\mathbf{a}_{11} = \frac{\gamma b \omega}{\gamma_0^2} K'_1(\underline{\sigma}b) \frac{K_1(\sigma_0 h)}{I_1(\underline{\sigma}h)} - \frac{\gamma b \omega}{\gamma_0^2} I'_1(\underline{\sigma}b) \frac{K_1(\underline{\sigma}b)}{I_1(\underline{\sigma}b)} \frac{K_1(\sigma_0 h)}{I_1(\underline{\sigma}h)}$$

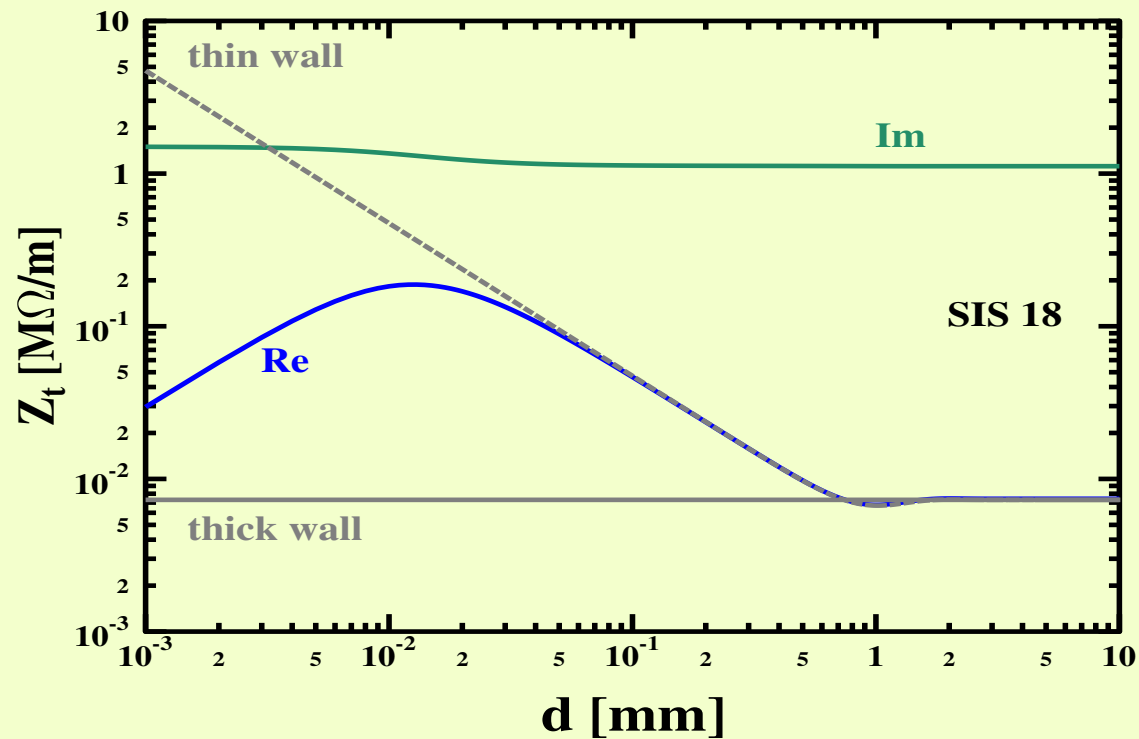
$$\mathbf{a}_{12} = \frac{\gamma b \omega}{\gamma_0^2} I'_1(\underline{\sigma}b) \frac{K_1(\underline{\sigma}h)}{I_1(\underline{\sigma}h)} \frac{I_1(\sigma_0 b)}{I_1(\underline{\sigma}b)} - \frac{\gamma b \omega}{\gamma_0^2} K'_1(\underline{\sigma}b) \frac{I_1(\sigma_0 b)}{I_1(\underline{\sigma}b)} - \frac{b \omega}{\gamma_0} I'_1(\sigma_0 b)$$

$$\mathbf{a}_{21} = \frac{\gamma h \omega}{\gamma_0^2} K'_1(\underline{\sigma}h) \frac{K_1(\sigma_0 h)}{I_1(\underline{\sigma}h)} - \frac{\gamma h \omega}{\gamma_0^2} I'_1(\underline{\sigma}h) \frac{K_1(\underline{\sigma}b)}{I_1(\underline{\sigma}b)} \frac{K_1(\sigma_0 h)}{I_1(\underline{\sigma}h)} - \frac{h \omega}{\gamma_0} K'_1(\sigma_0 h)$$

$$\mathbf{a}_{22} = \frac{\gamma h \omega}{\gamma_0^2} I'_1(\underline{\sigma}h) \frac{K_1(\underline{\sigma}h)}{I_1(\underline{\sigma}h)} \frac{I_1(\sigma_0 b)}{I_1(\underline{\sigma}b)} - \frac{\gamma h \omega}{\gamma_0^2} K'_1(\underline{\sigma}h) \frac{I_1(\sigma_0 b)}{I_1(\underline{\sigma}b)}$$

Transverse Impedance: SIS18

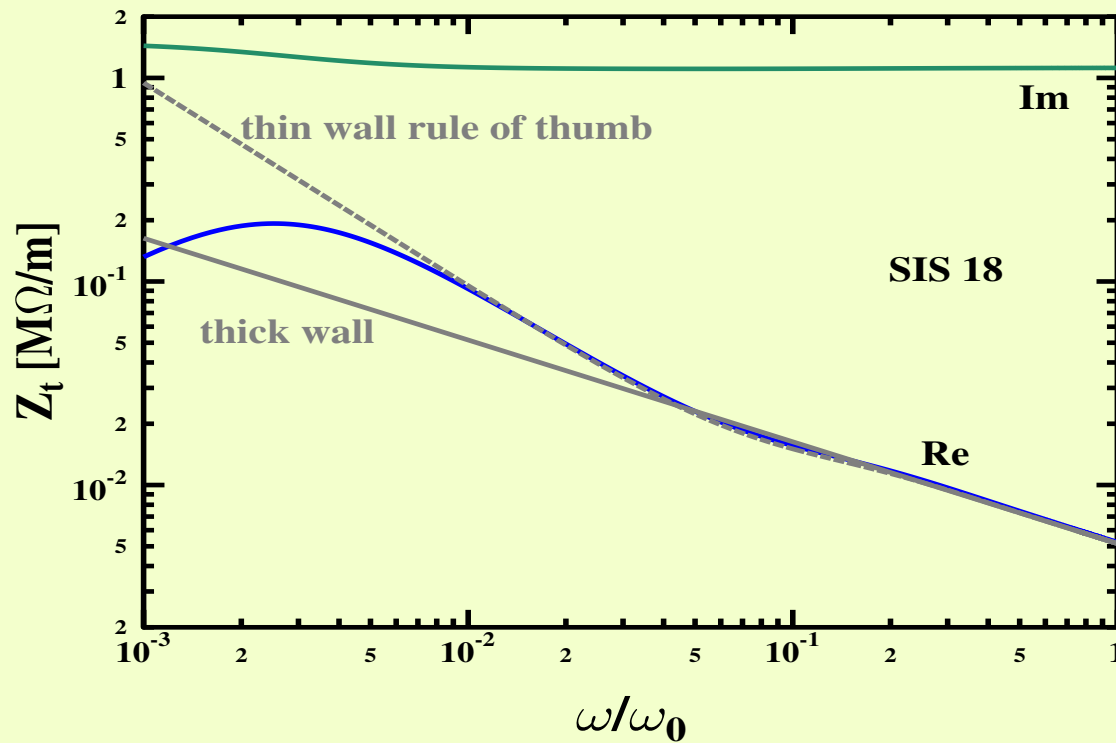
$C = 216$ m, $b = 10$ cm, $a = 0.5b$, $d_w = 0.3$ mm, $\gamma = 2$, $\omega = 0.5\omega_0$



Thin wall rule of thumb: $Z_{\perp,1} = \frac{2}{b^2 k_z} Z_{\parallel,0}$!!

Transverse Impedance: SIS18

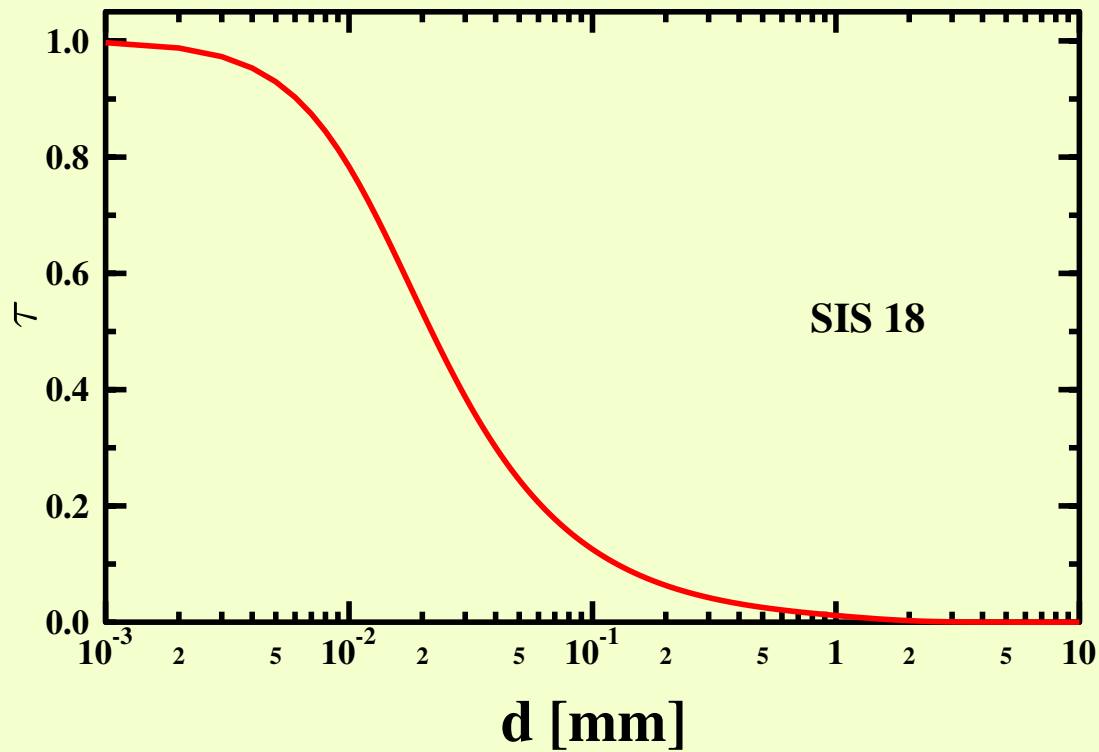
$C = 216$ m, $b = 10$ cm, $a = 0.5b$, $d_w = 0.3$ mm, $\gamma = 2$, $d = 2.5$ mm



Thin wall rule of thumb: $Z_{\perp,1} = \frac{2}{b^2 k_z} Z_{\parallel,0}$!!

Transmission Coefficient: SIS18

$C = 216 \text{ m}$, $b = 10 \text{ cm}$, $a = 0.5b$, $d_w = 0.3 \text{ mm}$, $\omega = 0.5\omega_0$, $\gamma = 2$



Summary

- Derivation of analytical expressions for Z_{\parallel} and Z_{\perp} for arbitrary frequency ω , energy γ and wall thickness d
- Beam shielding is of relevance for high-current SIS18 upgrade and new SIS100

Compromise: to reduce return currents to 1% of peak value d of few 0.1mm of St. Steel provides good return current shielding

- Both TM and TE modes contribute to the transverse resistive-wall impedance:
Positive total real part and Positive total imaginary part (Inductive)
- Apply Panofski-Wenzel theorem at the same azimuthal number
- **Needed:** Comparison between existing expressions (arbitrary d) from different authors for the transverse coupling impedance

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