A strong concern in FFAG design studies: codes
- Developements in Zgoubi -

Main issues:

1. “Scaling optics”: non-linear field, radially and axially, $B = B_0 \left( \frac{r}{r_0} \right)^K \ln \left( \frac{r}{r_0} / w - N \theta \right)$

2. “Non-scaling, linear optics”: reference orbit keeps moving ($x_{c.o.}(\delta p/p) \approx \text{arc of parabolla}$); in addition longitudinal motion is highly non-linear (fast acceleration, near-crest)

3. “Non-scaling, non-linear optics”: non-linear field, both radially and axially, fast acceleration

4. “Isochronous optics”: non-linear transverse field, fast acceleration

Use of first order optics codes is tedious, and of limited efficiency in designing types 1., 3., [4.] lattices: cannot (easily) model non-linear transverse behavior.

It is difficult to account for rapid geometrical variations of fields.

DA tracking and 6-D tracking difficult to perform in all 1.-4. cases, whereas it is a fundamental (and not necessarily just a final) stage in the design of FFAG machines
Review of Current FFAG Lattice Studies in North America
J. Scott Berg, Robert Palmer, Alessandro Ruggiero, Dejan Trbojevic et als.
Proc Cyclo. Conf. Tokyo, 2004

TRACKING RESULTS
Linear non-scaling FFAGs are a relatively new invention, and until recently, very little tracking has been done on them. One must carefully consider the nonlinear effects in these machines to find the dynamic aperture. The magnets are relatively short compared to their apertures, and thus end effects in the magnets become very important. So whatever tracking code is used must properly include these effects. COSY Infinity [11] has excellent built-in handling of magnet ends, but its use of truncated power series can at times (but not always) be problematic for the large energy acceptances required for FFAGs [12, 15]. ICOOL [16] has extensive facilities for handling end fields, as does ZGOUBI [17], and both of these codes have been used to do tracking for FFAGs [18, 19].
1 The Zgoubi method

Very good symplectivity, an good candidate for the challenge of tracking in FFAG rings!

\[ \text{Position : } \vec{R}(M_1) \approx \vec{R}(M_0) + \vec{u}(M_0) \Delta s + \vec{u}'(M_0) \frac{\Delta s^2}{2!} + \ldots + \vec{u}'''\ldots'(M_0) \frac{\Delta s^6}{6!} + \ldots \] (1)

\[ \text{Velocity : } \vec{u}(M_1) \approx \vec{u}(M_0) + \vec{u}'(M_0) \Delta s + \vec{u}''(M_0) \frac{\Delta s^2}{2!} + \ldots + \vec{u}'''\ldots'(M_0) \frac{\Delta s^5}{5!} + \ldots \] using \( \vec{u}' = \vec{u} \times \vec{B} \) (Lorentz equation), \( \vec{u}'' = \vec{u}' \times \vec{B} + \vec{u} \times \vec{B}' \), \( \vec{u}''' = \ldots \) etc.

![Diagram](image_url)

Figure 1: Position and velocity of a particle in the reference frame.

Note: if spin tracking, then in addition

\[ \vec{S}(M_1) \approx \vec{S}(M_0) + \vec{S}'(M_0) \Delta s + \ldots + S'''\ldots'(M_0) \frac{\Delta s^4}{4!} + \ldots \text{ using } \vec{S}' = \vec{S} \times \Omega(\vec{u}, \vec{B}) \] (2)

**NEEDED INGREDIENTS** : field and derivatives.
2 Simulation of an FFAG radial sector triplet

GEOMETRY/FIELD FOR ONE DIPOLE:

When alone, a dipole is encompassed in the magnetic field region defined by the angle \( \Delta \Gamma \). The dipole geometrical parameters yield the mid-plane vertical field:

\[
B_{z,i}(r, \theta) = B_{z0,i} \mathcal{F}_i(r, \theta) \mathcal{R}_i(r)
\]

by using

\[
\mathcal{R}_i(r) = (r/R_{0,i})^K
\]

and

\[
\mathcal{F}_i(r, \theta) = \mathcal{F}_{\text{Entrance}}(r, \theta) \times \mathcal{F}_{\text{Exit}}(r, \theta) \times \mathcal{F}_{\text{Lateral}}(r, \theta)
\]

simulating the effect of EFB’s —— discussed next slide.

GEOMETRY/FIELD FOR A SECTOR TRIPLET:

Basically, \( \text{N}=3 \) (here) dipoles are encompassed in the magnetic field region defined by the angle \( \Delta \Gamma \), they are positioned by \( \text{ACNi} \). Obtaining the total field is a matter of summation over the \( \text{N} \) neighboring dipoles (not more than a trick)

\[
B_z(r, \theta) = \sum_{i=1}^{N} B_{z,i}(r, \theta) = \sum_{i=1}^{N} B_{z0,i} \mathcal{F}_i(r, \theta) \mathcal{R}_i(r)
\]  

Derivatives:

\[
\frac{\partial^{i+j} \vec{B}_z(r, \theta)}{\partial \theta^i \partial r^j} = \sum_{i=1}^{N} \frac{\partial^{i+j} \vec{B}_{zi}(r, \theta)}{\partial \theta^i \partial r^j}
\]

(Note: this is a “trick”, we do not assume linear superposition)
Numerical results: Field along an arc of a circle in a radial sector:

\[ B_z(r, \theta) = \sum_{i=1,N} B_{zi}(r, \theta) = \sum_{i=1,N} B_{z0,i} \mathcal{F}_i(r, \theta) \mathcal{R}_i(r) \]

**MID-PLANE FIELD IN FFAG SECTOR TRIPLET:**

**OFF MID-PLANE, CLOSE TO POLE:**

![Graphs showing field distributions](image)

Figure 2: \( B_z(r_0, \theta, z) \) (Eq. 3) at traversal of the 30 degrees aperture FFAG triplet in the 50 MeV closed orbit region and for either \( z = 0 \) (left plot) or \( z = 6 \) cm as obtained by off mid-plane extrapolation (right). On both plots the thick curve represents the full field, as obtained by superposition of the separate contributions of each one of the three dipoles represented by the thin curves.

![Graphs showing derivatives](image)

Figure 3: Mid-plane derivatives \( \partial B_z(r, \theta) / \partial \theta \) (left plot) and \( \partial^2 B_z(r, \theta) / \partial \theta^2 \) (right) (Eq. 4) along a closed orbit (\( r \approx 4.87 \) m). *Note*: these plots show that the ray-tracing integration step \( \Delta S \) (and also the mesh size in the case field maps would be used to simulate such magnets) has to be small.
3 Application: the KEK 150 MeV proton FFAG ring

The field in the case of the geometrical method has been tuned so as to yield results as close as possible to the TOSCA map case. This was done using two constraints:

1/ closed orbit must be the same as in the TOSCA map case at 43 MeV (value chosen because it is about half-way between \( r_{\text{min}} \) and \( r_{\text{max}} \)),

2/ same horizontal and vertical tunes with both methods.

- Condition 1/ yields BF, and condition 2/ yields BF/BD ratio.
- \( \textit{Remark} \): no effort was made to have the analytical field shape closer to TOSCA case in the BD and drift regions. This would be possible by matching of the \( C_i \) fringe field coefficients.
Stability limits - of the order of 1 cm.rad acceptance!

The limits of stable motion for 5 energies, with better than \( \Delta r = \pm 0.1 \) mm accuracy.
Full acceleration cycle, from 12 to 150 MeV is now experimented.

Characteristics of the acceleration after [KEK Pubs.]:
- $\hat{V} = 19$ kV,
- $\phi_s = 20$ degrees

RF frequency after previous ray-tracing results: from 1.62 to 4.46 MHz.

Sample synchrotron motion. 2 particles: $r_0 = 4.48$ m and respectively $E=12$ MeV and 12 MeV+1%.

Figure 4: Acceleration from 12 MeV to 150MeV about (from 1.62 to 4.46 MHz), $\phi_s = 20$ degrees, $r_0 = r_{c.o.}(12 \text{ MeV})$, $z_0 = 3$ cm.

Horizontal phase-space (right) and vertical motion (vs. $r$); observation is at center of drift.
4 Application : phase rotation in PRISM

In PRISM, $p_0 = 68$ MeV/c ±20% (E ≈ 20 ± 7 MeV), to be brought down to less than ±%

Working hypothesis : the earlier design with 8 cells. Design/geometrical parameters as in publications, except for : only one cavity,

For simplicity just 5 particles are tracked, representative of the momentum span.

No optimization has been done, just want to show that the geometrical method for 3-D field simulation can be applied starightforwardly, from simply the geometrical parameters of the DFD triplet.

5 first turns (of a 400-turn tracking). The initial beam is the upright one in the dp-phi space, the solid line shows the final beam.

<table>
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<th>constant gap :</th>
<th>variable gap</th>
<th>(s(\alpha)(5/r)^3*6.3cm)</th>
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<td>beta_x / z</td>
<td>p/p0 1-Q_x / 1-Q_z</td>
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<td>.8 0.588170 / 0.698624</td>
<td>0.311 / 1.551.8</td>
<td>.8 0.609184 / 0.715739</td>
</tr>
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</table>
5 A non-scaling design: 8 to 20 GeV muon isochronous ring

Magnetic field in bd, BF and BD.
The lower plot shows the field in a cell along the 20, 14, 11, 9.5 and 8 GeV closed orbits.

G. Rees design.
Needs precision tracking: the isochronism has to be controlled at a $10^{-6}$ precision.

It means accuracy is necessary on
- the description of magnetic field in optical elements,
- ray-tracing.

\[ B_{bd}(x) = -3.45623 - 6.689211 \times x + 9.403200 x^2 - 7.623605 x^3 + 360.3808 x^4 + 1677.7968 x^5 \]
\[ B_{BF}(r) = -0.25776 + 16.62046 r + 29.73987 r^2 + 158.65762 r^3 + 1812.1753 r^4 + 7669.5302 r^5 \]
\[ B_{BD}(x) = 4.22034 - 9.65952 x - 45.4722 x^2 - 322.1230 x^3 - 5364.3096 x^4 - 27510.421 x^5 \]
Stability limits

Figure 5: 1000-cell, stability limits of pure horizontal motion, at better than 0.1 cm precision.

Figure 6: 1000-cell or more, vertical motion stability limits, at better than 0.1 cm precision.
Amplitude detuning

Figure 7: Amplitude detuning. Left: pure radial motion, with for each energy, $x_0$ varied from closed orbit position to maximum stable amplitude. Right: axial motion, with for each energy, $z_0$ varied from zero to maximum stable amplitude while $x \equiv x_{co}$, always.