Coherent Beam-Beam Modes in the LHC for Multiple Bunches, Different Collision Schemes and Machine Symmetries

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Abstract

In the LHC almost 3000 bunches in each beam will collide near several experimental regions and experience head-on as well as long range beam-beam interactions. In addition to single bunch phenomena, coherent bunch oscillations can be excited. Due to the irregular filling pattern and the unsymmetric collision scheme, a large number of possible modes must be expected, with possible consequences for beam measurements. To study these effects, a simulation program was developped which allows to evaluate the interaction of many bunches. It is flexible enough to easily implement any possible bunch configuration and collision schedule and also to study the effect of machine imperfections such as optical asymmetries. First results will be presented and future developments are discussed.

INTRODUCTION

The spectra of the barycentric motion and the mode frequencies of coherent beam-beam modes are well known and understood for the case of a few bunches colliding head-on [1, 2]. Present and future colliders have many bunches and multiple interaction points and a much richer spectrum of modes must be expected [3]. This is in particular true when the collision points are not symmetrically distributed and additional effects due to non-symmetric collision schemes [4, 5] or asymmetric colliders like two-ring schemes [6] must be expected.
In the LHC there are a number of effects which break the symmetry between the collision points:

- Asymmetric configuration of the collision points
- Presence of a large number of parasitic long range interactions
- Unavoidable PACMAN effects [7, 8]
- It is impossible to make the bunches collide exactly head-on [9, 10].

In the case of multiple head-on collisions these modes can be analyzed with a linearized model searching for the eigenmodes of the full single turn map. However, when the non-linear long range interactions are included, the linearized treatment is not adequate. One therefore might expect a fairly large number of modes which may obscure tune measurements or feedback systems. The presence of a large number of modes due to the effect of local, parasitic interactions was already studied in [11, 12] but without possible PACMAN effects and for a simplified LHC collision scheme.
It is therefore important to define possible configurations which minimize the number of modes and provide cleaner spectra.
For the evaluation a strong-strong simulation program is written, using a rigid Gaussian model for the bunches.

SIMULATION PROGRAM

The simulation program must allow to:

- Track each bunch of both beams independently around the ring
- Apply head-on and long range interactions at bunch encounters
- Give initial kicks to single bunches or a range of bunches to simulate excitation (e.g. for tune measurement)
- Analyze the motion of selected or a range of bunches
- Show the complete set of possible coupled beam-beam modes

In order to evaluate different scenarios, the program must be very flexible to allow easy changes of parameters such as tunes, number of bunches, filling scheme, collision scheme etc. In particular it must allow different crossing planes.
The possibility to change the phase advance between collision points is important.
Statistical fluctuations such as bunch intensity, emittance etc. must be possible to simulate.
It should be possible to simulate and demonstrate PACMAN effects.
In order to get all correct modes of the bunches coupled by head-on and long range interactions, all individual interactions must be simulated in full. In particular, lumping several long range interactions is therefore not adequate.
For future extensions it must be possible to add multi-particles to replace rigid bunches in a straightforward way.

Parameters

To describe the motion of a rigid bunch the following parameters are used:

- Horizontal position and angle of barycentre: X and X'
- Vertical position and angle of barycentre: Y and Y'
- Horizontal position and angle of single particles: x and x'
- Vertical position and angle of single particles: y and y'
- Longitudinal phase (or position s) and energy deviation: ϕ (or s) and δ

For extension and later use, following parameters are foreseen and stored:

- Bunch intensity (to determine beam-beam kick)
- Bunch emittance (to determine beam-beam kick)
- Tune shift ΔQ_X and ΔQ_Y with respect to a nominal bunch.

Input description

For the simulation it is necessary to describe the arrangement of the bunches around the machine and their possible interactions with other bunches or machine elements. For simplicity it must be optimized to study beam-beam interactions. However, the description should be very flexible to allow the study of different filling or collision schemes as well as optical properties of the machine. I have followed the strategy designed for beam-beam tracking and the computation of self-consistent properties [9, 10, 13, 14] and included all the necessary extensions.

Bunches in the ring and description of filling scheme
The description of the bunch filling scheme is given in the form of $groups$. Each group has two parameters: the first specifies the number of slots n and the second whether the n slots are occupied by a bunch (1) or whether the slots are empty (0). The total number of slots must be equal to the machine circumference devided by the bunch spacing. It is therefore vital that all empty slots are defined as well as all filled slots. The number of groups per line is specified at the beginning of the description file. To define 1 bunch followed by 39 empty slots one could use:

```
# bunch filling example 1
#Number of groups
2
 1 1 39 0
 1 1 39 0
 1 1 39 0
 1 1 39 0
```

This example describes 4 equidistant bunches spread out in 160 slots (possible bunch positions) while the example below represents the actual LHC bunch filling scheme [7, 18, 19].

```
# bunch LHC filling example
# number of groups
8
72 0  8 0  72 1  8 0  72 1   8 0 30 0   0 0
72 1  8 0  72 1  8 0  72 1   8 0 30 0   0 0
72 1  8 0  72 1  8 0  72 1   8 0 72 1  39 0
72 1  8 0  72 1  8 0  72 1   8 0 30 0   0 0
72 1  8 0  72 1  8 0  72 1   8 0 30 0   0 0
72 1  8 0  72 1  8 0  72 1   8 0 72 1  39 0
72 1  8 0  72 1  8 0  72 1   8 0 30 0   0 0
72 1  8 0  72 1  8 0  72 1   8 0 30 0   0 0
72 1  8 0  72 1  8 0  72 1   8 0 72 1  39 0
72 1  8 0  72 1  8 0  72 1   8 0 30 0   0 0
72 1  8 0  72 1  8 0  72 1   8 0 30 0   0 0
72 1  8 0  72 1  8 0  72 1   8 0 72 1  39 0
```

The number of slots in this case is 3564 which is one tenth of the LHC harmonic number. The description should maximize the readability, although any format is possible.

Positions and actions
When one is interested in beam-beam interactions, only every half bunch spacing something can happen (i.e. where two bunches from the two beams could meet). For N slots defined by the filling scheme (i.e. number of possible bunch positions), one has 2N positions where such actions can occur. In the description the numbering of the positions goes from 1 to 2N in the direction of the clock-wise beam.

Definition of actions
At any position, an action can be requested for a bunch when it is in that place. For beam-beam interactions (head-on or long range) two bunches (i.e. one from each beam) must be at this position. The different actions are specified by a code number. Possible actions are:

- Head-on collision (at the specified position, code 2 or -2)
- Head-on and long range collisions (left and right of a specified head-on collision)
- Multiple long range collisions (left and right of a specified position, code 4 or -4)
- Single separated collision (code 5 or -5)
- Linear matrix transfer of a bunch (code 3)
- No action (default)

Additional actions, e.g. non-linear elements or correction devices, can easily be defined.

Head-on collision:

The code for a head-on collision point is either **2** or **-2**. The positive sign indicates horizontal and the negative sign vertical separation of the associated long range interactions, i.e. crossing plane in the case of the LHC. The strength of the head-on collision is determined by the beam-beam parameters which is either taken from the general input file or calculated from the bunch intensities, emittances and positions of the two colliding bunches. Before and after a head-on collision, the bunches are advanced in transverse phase space by $\pi/2$.

Long range collisions left and right of a head-on collision:

When a head-on collision point is defined like above, a number of long range collisions left and right of the collision point can be specified on the action statement for the head-on collision by specifying the number of collision points, i.e. the number of positions where long range interaction can occur. E.g. the line:

```
161  2  -15  +15
```

specifies a head-on collision at position 161 with horizontal crossing and 15 long range interactions on each side.

Long range collisions left and right of a specified position:

When an action code of **4** or **-4** is specified, only the long range interactions left and right of a specified position are active, the central head-on collision is ignored. This can be used to simulate a crossing angle configuration when the central head-on collision point is separated and the bunches experience long range interactions left and right of the symmetry point. However the rotation by $\pi/2$ before and after the specified position is performed to ensure the correct phase relationship between the long range interactions before and after.

Separated collisions

An action code of **5** or **-5** is used for a single separated interaction (e.g. in a Pretzel scheme). The third and fourth parameters are ignored.

Linear transfer of the bunches:

With the action code **3** a linear transfer is defined. The two parameters are used to control the phase advance of the transfer. The parameters specify the phase advance in units of 2π (tune). The phase advance between any point in the machine and in particular between interaction points can easily be controlled that way. The two rotations of $\pi/2$ for each head-on interaction point must be taken into account to get the correct overall tune.
In the present implementation the phase advance between two points in the machine is assumed to be the same for the forward and backward beams. In a two ring machine like the LHC this is not always the same.

Description of collision scheme

The collision scheme defines the actions to be performed at the possible positions. This description is an extension of the scheme defined for [13]. Every action consists of one line which defines first the position of the action, the second column is the code of the desired action and the third and fourth columns are parameters required by the action. Typical collision descriptions are:

```
#Collision scheme 1 (for filling example 1):

   1   2   -5  +5
  21   3   7.535  6.91375
  41  -2   -5  +5
 101   3   23.605 21.74125
 161  -2   -5  +5
 221   3   23.605 21.74125
 281   2   -0  +0
 301   3   7.535  6.91375
```

which defines 4 collision points where three have long range collisions on both sides of the head-on collision points. The machine has an eightfold symmetry in geometry and phase advance. As a further example, a collision scheme representing the LHC with its present filling scheme and layout of the four experiments is shown below.

```
#Collision scheme LHC (for LHC filling scheme):

   1  -2   -15  +15
 447   3    8.046   6.940
 892  -2   -0  +0
2229   3   23.015  21.821
3565   2   -15  +15
4902   3   23.533  20.689
6235   2   -0  +0
6684   3    7.716   7.870
```

Since the filling scheme defines the number of bunches and positions, the collision definition scheme must always follow the definition of the filling pattern.

Parameter input

At the start of the program, a parameter file is read in to define the basic input data. The name of this file is taken as a command line argument of the program.

```
collision:  coll_ref.in        // input collision scheme
filling:    fill_ref.in        // input filling scheme
use bunch:   1                 // define bunch for analysis
number of turns: 14            // number of turns: 2**14
bunches to kick:   5           // kick 5 consecutive bunches
sigma intensity: 0.8     // random or systematic intensity
                                             fluctuations
beam-beam parameter: 0.0025       // beam-beam parameter
```

Actions

Linear transfer

At a position requiring a linear transfer I use a linear transfer map:

$$\begin{pmatrix} X \\ X' \end{pmatrix}_{n+1} = \begin{pmatrix} \cos(\Delta\mu_X) & \sin(\Delta\mu_X) \\ -\sin(\Delta\mu_X) & \cos(\Delta\mu_X) \end{pmatrix} \begin{pmatrix} X \\ X' \end{pmatrix}_n \quad (1)$$

as expressed for example for the horizontal coordinates where $\Delta\mu_X$ is taken from the input files (i.e. collision scheme). The same transfer map with $\Delta\mu_Y$ is applied to the vertical coordinates Y and Y'.

Head-on beam-beam interaction

To calculate the head-on beam-beam kick on a bunch, the counter-rotating beam distribution is assumed to have a Gaussian density distribution in the two planes with barycentres at (X^*, Y^*) and squared transverse sizes $\Sigma^*_{xx} =< (x-X)^2 >^*$ and $\Sigma^*_{yy} =< (y-Y)^2 >^*$. In that case the beam-beam force can be expressed analytically. The * denotes parameters of the opposing beam. In the case of rigid bunches the transverse sizes are kept constant. We apply a horizontal beam-beam kick at the IP (equivalent for the vertical beam-beam kick):

$$\Delta X' = \frac{2r_p N^*_p}{\gamma}\frac{\beta_x}{\sigma^2_x} F_x(X - X^*, Y - Y^*, \Sigma^*_{xx}, \Sigma^*_{yy}) \quad (2)$$

with r_p the classical proton radius, N^*_p the bunch population (* indicates parameters of the counter-rotating beam), γ is the relativistic Lorentz factor, β_x the horizontal betatron function at the IP, σ_x the horizontal rms size and F_x (or, equivalently, F_y for the vertical beam-beam kick) given by

$$\begin{gathered} F_{\{x,y\}}(X - X^*, Y - Y^*, \Sigma^*_{xx}, \Sigma^*_{yy}) = \\ \frac{\{X, Y\}}{(X^2+Y^2)}\left[1 - \exp\left(-\frac{X^2+Y^2}{\Sigma^*_{xx}+\Sigma^*_{yy}}\right)\right]. \end{gathered} \quad (3)$$

which is the expression for round beams when $\Sigma_{xx} \approx \Sigma_{yy}$. When the beams are not round, we use the Bassetti-Erskine formula for the evaluation of the kick [22]. In the horizontal plane. The map at the beam-beam interaction is then:

$$\begin{pmatrix} X \\ X' \\ Y \\ Y' \\ s \\ \delta \end{pmatrix}_{n+1} = \begin{pmatrix} X \\ X' + \Delta X' \\ Y \\ Y' + \Delta Y' \\ s \\ \delta \end{pmatrix}_n \quad (4)$$

The beam-beam parameters are defined by

$$\xi_{\{x,y\}} = \frac{N_p r_p \beta_{\{x,y\}}}{2\pi\gamma\sigma_{\{x,y\}}(\sigma_x + \sigma_y)} \quad (5)$$

With the nominal LHC parameters we have $\xi \approx 0.0034$.

Before and after each head-on collision, I apply a phase advance of π/2 in each plane.

Long range beam-beam interaction

For the calculation of the long range beam-beam kick, the expressions for the head-on interaction must be modified to take the separation into account. The constant part of the kick in the plane of separation must be subtracted. Assuming a constant horizontal separation d and using the expression (3):

$$\Delta X' = \frac{2r_p N^*_p}{\gamma}\frac{\beta_x}{\sigma^2_x}\left[F_x(X + d - X^*, Y - Y^*, \Sigma^*_{xx}, \Sigma^*_{yy})\right] \\ \left[F_x(d, 0, \Sigma^*_{xx}, \Sigma^*_{yy})\right] \quad (6)$$

one gets the deflection $\Delta X'$ and for the other plane we have:

$$\Delta Y' = \frac{2r_p N^*_p}{\gamma}\frac{\beta_y}{\sigma^2_y}\left[F_y(X + d - X^*, Y - Y^*, \Sigma^*_{xx}, \Sigma^*_{yy})\right] \quad (7)$$

Tracking strategies

Initial conditions

The barycentres of the bunches of the two beams can be set all to zero at the start of the program or distributed according to a Gaussian distribution. Furthermore, a single bunch or a small number of bunches can be excited at the beginning, simulating e.g. a tune measurement. This is specified in the input files.

Rotation of bunches in both rings

Beam 1 bunches travel increasing number of positions, beam 2 bunches decreasing number of positions.
At each step, every bunch is advanced by one position, i.e. half a bunch spacing. One complete turn in the machine therefore requires 2N steps.
The calculations for all bunches of a beam at each step are independent and it can be envisaged to make use of parallel processing, in particular when the bunches consist of many macro-particles in a later version of the program.

Data processing

By a Fourier analysis of the barycentre of the bunches, as calculated turn by turn, we obtain the tune spectra of the dipole modes. For only one bunch per beam the two spectra of the two bunches are equivalent. For more than one bunch per beam the spectra of bunches with the same collision scheme are also equivalent. Analyzing the sum $(X^{(1)} + X^{(2)})$ or the difference $(X^{(1)} - X^{(2)})$ of the barycentre of two colliding bunches of the two beams (denoted by (1) and (2)) show the spectra of the 0- and π-modes separately. This is useful to analyze the details of the modes.

Program validation

Head-on effects

To validate the program, I have simulated two head-on collisions in two interaction points, opposite in azimuth. A value of 0.0025 was used for the linear beam-beam parameter. The results are shown in Figs. 1 and **??**. Equal charges of the two colliding beams are assumed and the frequencies are therefore shifted downwards from the unperturbed tunes. In Fig. 1 the spectrum of the first bunch of beam 1

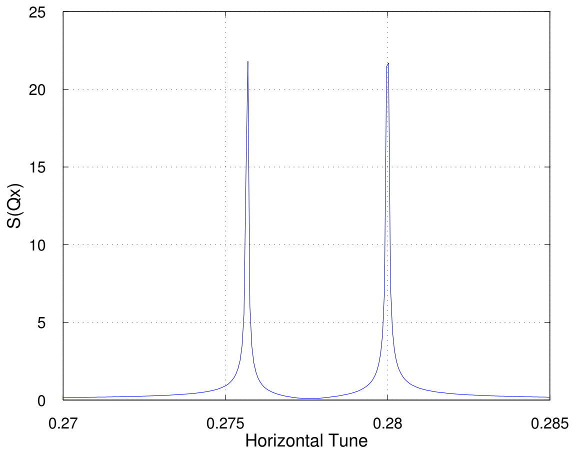

Figure 1: Symmetric Head-on collisions in IPs 1 and 5.

is shown; the spectra clearly shows the two coherent beam-beam modes. The sum signal of the two bunches gives the so called 0-mode (right peack in Fig. 1) while the difference signal gives only the π-mode signal (left peak in Fig. 1). This is in agreement with the expectations. The frequency shift between the 0-mode and the π-mode is however not correct in a rigid bunch model and the forces must be calculated from the real field distribution [16, 17]. For the purpose of this report to study the spectra of dipole oscillations the rigid Gaussian model is adequate.

RESULTS

With the available simulation program, the following effects can be studied and the dependence of the results on the optical and collision configuration can be evaluated.

- Head-on interactions only (one bunch per train or no long range positions)
- Head-on and long range interactions (multiple bunches per train)
- Excitation of single and multiple bunches in a train for measurement purposes

Multiple head-on interactions

In the case of multiple head-on collisions in a machine, the symmetry properties of the layout are very important for the spectra. A high degree of symmetry can lead to the degeneracy of modes, i.e. identical frequencies, and their suppression in the spectra. Breaking the symmetry by choosing a non-symmetric collision scheme or phase advance differences between the interaction points may cancel this effect and leads to the appearance of additional modes in the spectra. In the following I assume a collider with an eightfold symmetry of the possible collision points and number the interaction regions from 1 to 8. In this case the interaction points 1 and 5 are opposite in azimuth. This resembles the geometrical layout of the LHC straight sections. The Fig. 1 shows two head-on collisions opposite in azimuth with symmetric optical layout. The

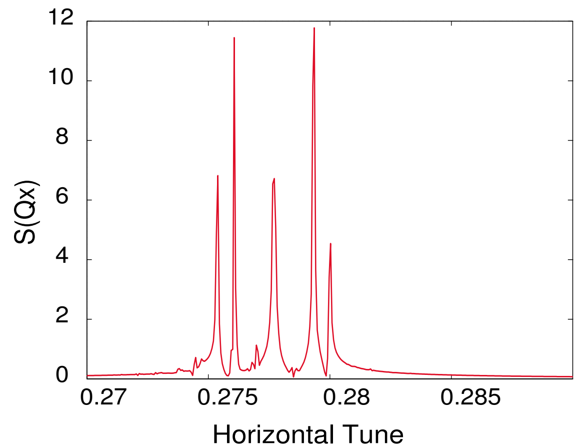

Figure 2: Non symmetric head-on in IPs 1 and 2.

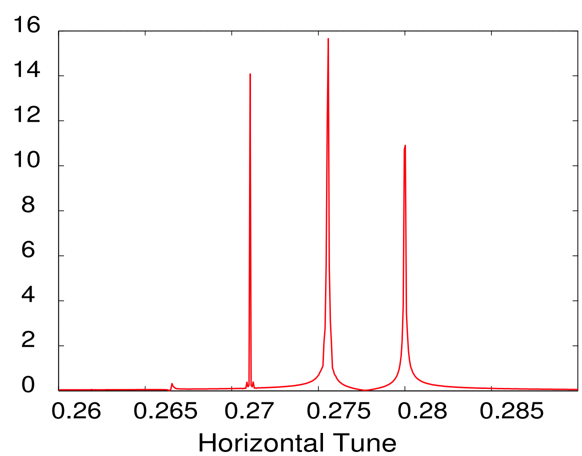

Figure 3: Symmetric head-on in IPs 1, 3, 5 and 7.

effect of additional head-on collisions and non symmetries in the optical layout are shown in the horizontal spectra in Fig.2 and in Fig.3 are shown for two simple cases. It can be observed that a higher degree of symmetry (or periodicity) leeds to degeneracy of mode frequencies and fewer spectral lines. This confirms earlier findings [4, 5, 11, 12] and the importance of symmetries for coherent modes. The number of lines in the spectra can be qualitatively understood by analysing the collision pattern of the bunches. The number is closely related to the number of bunches to which the measured bunch couples directly or indirectly (i.e. via other bunches). For example this explains the number of spectral lines when collisions occur only in interaction points 1 and 2 (Fig.2) and the reduced number when collisions occur in points 1, 3, 5 and 7 (Fig.2).

Collisions with the LHC interaction region layout

The collision scheme of the LHC with its four interaction regions was illustrated as an example already. Although the geometry has an eightfold symmetry, the phase advances between the interaction points break this symmetry and we must expect a richer spectrum of modes.

LHC interaction region layout with standard phase advance

The standard LHC collision scheme was already used as an example before. For the tracking studies, the arcs can be compressed since no action can happen except a single linear transfer. The number of bunches is reduced to 9 per train and to observe PACMAN effects, the number of long range positions is 5 on each side of the collision point. This will strongly reduce the required computing time but has no qualitative effect on the results. The nominal collision definition scheme used in the simulation is then:

```
  1   2   -5  +5
 41   3    8.046   6.940
 81  -2   -0  +0
202   3   23.015  21.821
321  -2   -5  +5
441   3   23.533  20.689
561   2   -0  +0
601   3    7.716   7.870
```

together with a filling scheme:

```
#Number of groups
2
 9 1 71 0
 9 1 71 0
 9 1 71 0
 9 1 71 0
```

LHC interaction region layout with symmetry between IP1 and IP5

Starting from the scheme above, it can be partially symmetrized to fulfill:

$$\Delta Q_x^{1\to 5} = \Delta Q_x^{5\to 1} = Q_x/2 \quad (8)$$

and we use:

```
  1  -2   -0  +0
 41   3   8.046  6.940
 81  -2   -0   +0
201   3  23.109 21.720
321   2   -0  +0
441   3  23.439 20.790
561   2   -0   +0
601   3   7.716  7.870
```

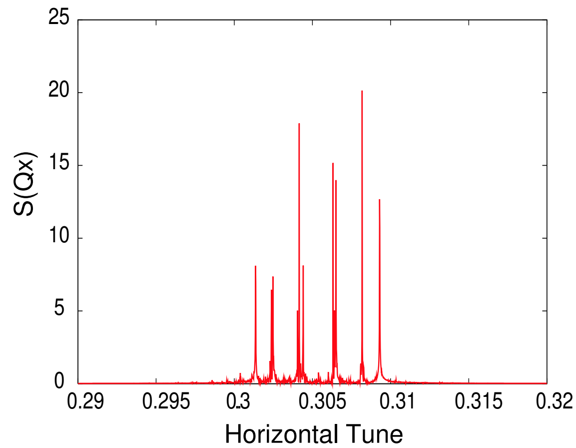

Figure 4: Head-on collisions in IPs 1, 2, 5 and 8 with nominal LHC phase advance between interaction points.

A further improvement is possible by adjusting the phase advance between interaction points 2 and 8 as shown below.

```
  1  -2   -0 +0
 41   3   8.046    6.940
 81  -2   -0  +0
201   3  23.109   21.720
321   2   -0 +0
441   3  23.6235   21.270
561   2   -0  +0
601   3   7.5315    7.390
```

The spectra for such a scheme are shown in Fig.5. The comparison between Fig.4 and 5 shows the effect of the symmetry between interaction points 1 and 5. Although the number of modes is not really changed, it must be expected that the Landau damping of modes with frequencies just below the 0-mode will "clean" the spectra around the 0-mode, i.e. the nominal tune, and therefore simplifies the tune measurements.

LHC interaction region layout with full eightfold symmetry

The fully symmetric version with eightfold symmetry in the phase advances is:

```
  1  -2   -0  +0
 41   3   7.78875  7.165
 81  -2   -0    +0
201   3  23.36625 21.495
321   2   -0  +0
441   3  23.36625 21.495
561   2   -0    +0
601   3   7.78875  7.165
```

The spectra for the fully symmetric machine are very similar to those obtained with the "tuned" collision scheme shown in Fig.5.

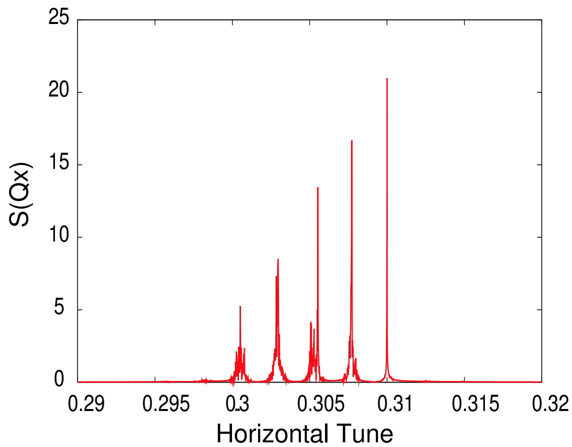

Figure 5: Head-on collisions in IPs 1, 2, 5 and 8. Phase advance symmetry restored between IP1 and IP5 and adjusted between 2 and 8.

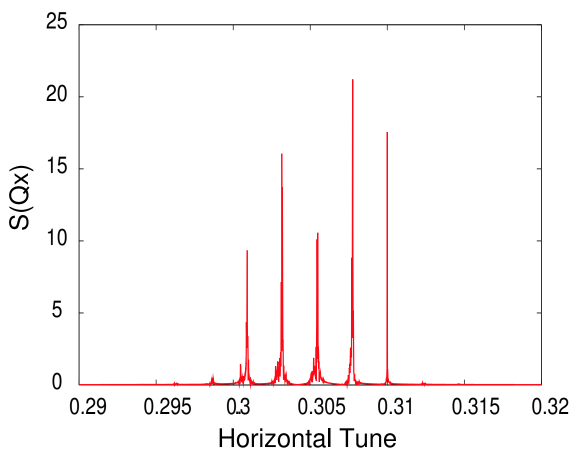

Figure 6: Head-on collisions in IPs 1, 2, 5 and 8 with full eightfold symmetry of phase advances.

SUMMARY

We have used a multi bunch simulation to compute the spectra of dipole oscillations driven by head-on and long range beam-beam interactions. The spectra largely depend on these interactions and the main observations can be summarized:

- Configuration of collisions should be symmetric to reduce number of dipole modes.
- Phase advance between low β interaction regions should be symmetric to allow degeneracy and compensation of coherent modes.
- Although not required for the compensation of first order PACMAN effects ([20]) or suppression of resonances ([5]), some flexibility of the phase adjustment between interaction points is desirable.
- Measurement should be on a single bunch following an excitation of this bunch if possible.

These should serve as recommendations when it becomes important to keep the spectra clean.
However, damping effects such as Landau damping due to the incoherent tune spread are not included in the present rigid bunch model and will be studied in the future using a multi-bunch, multi-particle simulation. It must be expected that these damping effects suppress a significant number of modes, in particular in the immediate neighbourhood of the 0-mode.

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