

BETA USERS' GUIDE

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1. INTRODUCTION

BETA is a computer code for use in the design and analysis of synchrotrons and storage rings. The program is working in an interactive mode and most of the output data are displayed in the form of plots. Interfaces for graphic packages GKS, Minigraf and Matlab are available.

The program performs the following calculations:

- linear lattice parameters calculations: if R.F. cavities or energy losses are explicitly described in the machine, BETA computes fully coupled (synchro-betatron) motion of particles. Otherwise it deals with betatron motion, with eventually horizontal-vertical coupling. The relevant parameters of the ring (first-order matrix, Twiss functions, dispersion function, betatron and synchrotron tunes, tune shifts as a function of betatron amplitude) are displayed, and optical functions may be plotted;
- In the case of an electron machine, the damping partition numbers, the damping times, beam emittances, bunch length and energy spread are also displayed;
- linear lattice matching;
- calculation of the second-order transfer matrix;
- calculation of the horizontal and vertical chromaticities and adjustment to any value;
- minimization of driving terms of third-order resonances induced by sextupoles and minimization of linear tune shifts with emittances;
- particle tracking including a Fourier analysis of the turn by turn position of the tracked particle and calculation of its frequency spectrum; synchrotron motion may be included;
- dynamic aperture calculation for on-momentum and off-momentum particles; results are obtained either for a fixed displaced energy or with synchrotron oscillations; the calculation is also performed in the presence of errors;
- simulation of multipole field errors;
- simulation of closed orbit errors;
- treatment of linear and non-linear effects of wigglers and undulators on beam dynamics;

The program uses the 7×7 matrix formalism (7×35 in the case of second-order transfer)[1]. For particle tracking, the treatment is performed in the following way:

- the traversal of standard elements such as drift spaces, dipoles, quadrupoles is described by the cumulative matrix resulting of the product matrix of the individual 7×7 matrices of the elements;
- when a sextupole is encountered, its second-order 7×35 matrix is used; therefore, sextupoles have to be defined as thin lenses in order to keep the tracking symplectic;
- multipoles are treated in the thin lens approximation. A kick corresponding to the multipole strength scaled with energy is applied;
- for an insertion device (wiggler or undulator), each period is divided into 10 slices; the field is integrated over each slice and the corresponding kick is applied at the middle of the slice;
- cavities are considered as thin lens elements; when a cavity is encountered, a kick in energy is given, according to a sinusoidal RF field.

The code is run by a sequence of commands which give access to different options. In each option BETA asks for specific input data which can be given in free format. Each command is identified by a keyword. Most of the options (starting with **FI**) can be executed in an arbitrary order provided that the necessary data have been updated.

A lot of new options have been included from the version 3.00 upwards. The main change concerns the integration of the program BEAM into BETA. It now appears as mode "BEAM" and allows among other options to toggle between a linear structure and a periodical structure. Furthermore kickers are no longer scaled in the tracking from version 3.09 upwards. The manual was also updated because of several new options.

2. CONVENTIONS

2.1 Reference system

The structure is described as a sequence of magnetic elements along a reference orbit. The reference orbit is a path of a charged particle having the central design momentum of the ring through idealized magnets with no fringing field (see Figure 1).

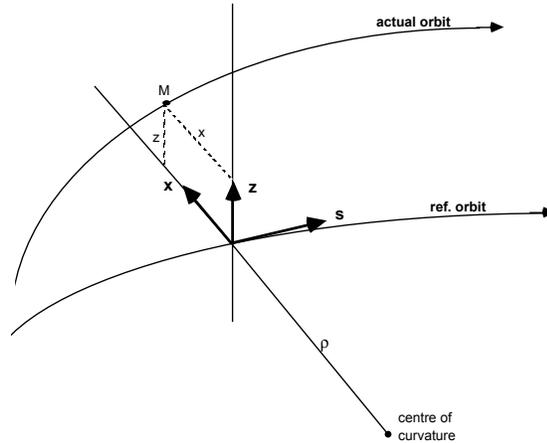


Figure 1: coordinate system

2.2 System of coordinates used in BETA

The reference consists of a series of straight lines and arc of circles. All magnets are assumed to be perfectly aligned on the central trajectory. The position of a given particle M is measured with respect to the curvilinear right-handed coordinate system (x,z,s). The two coordinates which are perpendicular to the reference orbit(s) are labelled x (horizontal) and z (vertical).

At any position, an arbitrary particle is represented by a single column matrix X, whose components are the positions, angles, and momentum of the particle with respect to the reference orbit, i.e.

$$\begin{pmatrix} x \\ x' = \frac{dx}{ds} \\ z \\ z' = \frac{dz}{ds} \\ l \\ \delta = \frac{\Delta p}{p} \\ 1 \end{pmatrix}$$

The 7th component of the vector X is used to represent the effect of a kick on the trajectory. When dealing with second-order formalism, the vector X is extended by the addition of the second-order components (see p. 17)

2.3 Sign conventions

The program uses the following sign conventions for the input data:

- Quadrupole: a strength $K > 0$ corresponds to horizontal focusing;
- Sextupoles: a strength $K > 0$ corresponds to horizontal focusing;
- Cavity: a negative value of the voltage V is required;

- Dipole: if the angle θ and the radius of curvature ρ are both positive, the dipole bends a particle to the right, i.e. towards negative x values. If $\theta < 0$ and $\rho < 0$, the particle is bent to the left;
- Edge focusing: see Figure 2 for ε and C sign conventions in the case of entrance pole face. Conventions are the same for exit pole face.

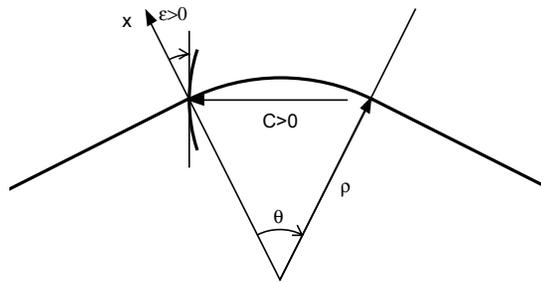


Figure 2 : Dipole conventions

3. INPUT FORMAT

Input for BETA is free format. To make the interactive use of BETA easier, the program often proposes predefined answers in rectangular brackets. These proposals can be changed by typing the desired answer after the colon. The line is sent to the computer by typing "ENTER", but only the part following the colon is decoded. In case of a simple "ENTER" after the colon the default value is taken. Different parameters on the same line (keywords or numerical values) must be separated by spaces, commas or slashes. Numerical parameters must be entered like FORTRAN constants (100, 100., 1.0E2 are equivalent) and keywords may appear either in uppercase or lowercase letters. When a list of options is proposed, the default answer ("Enter" key only) is the first option of the list.

4. OUTPUT FORMAT

Here we give a brief description of the output resulting of the transfer in the structure.

4.1 First-order matrix

The 7 x 7 transfer matrix for one superperiod is displayed. In addition to the usual six-dimensional matrices (x, x', z, z', l, δ), the 7th column is used to compute the effects of kicks. Energy dependent terms are computed for a default value of γ equal to 1. The actual γ value is taken into account by means of the definition of a cavity element (see option DE or MO).

The superperiod length and the mean radius of the machine are displayed.

4.2 Beam matrix

The Twiss parameters are printed via the standard beam matrix, as used in beam lines transport [2]:

$$\begin{aligned} \beta_x &= \sigma_{11} & \beta_z &= \sigma_{33} \\ \alpha_x &= -\sigma_{12} & \alpha_z &= -\sigma_{34} \\ \gamma_x &= \sigma_{22} & \gamma_z &= \sigma_{44} \end{aligned}$$

In the presence of coupling, BETA displays the three different 6x6 beam matrices corresponding to the independent invariants (or two 4x4 matrices if there is no cavity in the structure) (see appendix D).

4.3 Tunes

The fractional part of the betatron tunes (ν_x, ν_z) and the synchrotron tune (ν_s) are extracted from the 6x6 transfer matrix (coupling taken into account) and displayed. Then, the betatron tunes (including the integer part of the

tune) are computed.

4.4 Closed orbit

The first order closed orbit (x, x', z, z', l, δ) is displayed (6-dimensional if there is a cavity present in the structure, 4-dimensional if not). The chromatic closed orbit ($\eta_x, \eta'_x, \eta_z, \eta'_z$) is also computed as a result of the first-order matrix analysis.

4.5 Momentum compaction

The momentum compaction α , the η value ($\eta = \alpha - 1/\gamma^2$), the particle energy and γ are displayed. The default values are given for an electron machine running at the rest energy. To run at the desired energy, a cavity element must be included in the single elements list (see option DE or MO).

4.6 Damping partition numbers, damping times, equilibrium emittances

For an electron machine, BETA displays the synchrotron radiation integrals, the energy loss per turn, the damping partition numbers [3] J_x, J_y, J_e and the corresponding damping times. The natural horizontal emittance [4,5] is computed at the running energy (see appendix D). The energy spread and the bunch length (if a cavity is included in the structure) are displayed.

4.7 Driving terms and tune shifts with amplitude

The driving terms D, G, DG, F, L, H related to the third-order resonances excited by sextupoles are displayed as follows:

- the sum over 200 harmonics;
- the strongest harmonic;
- the corresponding resonance;
- the ratio between the strongest harmonic and the sum of all harmonics;

Tune shifts as a function of emittance are a combination of these driving terms. For more details on the formalism used, see Appendix A.

4.8 Sextupoles and quadrupoles

For each sextupole and quadrupole, the integrated strength (HI or KI), the azimuth along the machine θ , the β values and the betatron phases φ are displayed.

4.9 Second-order transfer

When a second-order transfer is performed, additional characteristics are displayed:

- the chromaticities defined as $\xi_{x,z} = (\Delta v_{x,z} / v_{x,z}) / (\Delta p / p)$;
- the variations of twiss parameters with momentum;
- second-order coefficients of the transfer matrix.
- One details about second order transfer are given in option **MO**, section 5.

5. INPUT FILE

At the beginning of each BETA session the user usually reads a file (command **FI**) which contains the lattice structure and has "str" as extension of the filename. It is built up of several sections each separated by a line of stars eventually containing a keyword as a title of the section. The following sections exist:

- beam matrix¹ : The lower half of the beam matrix is given followed by the values for the horizontal and the vertical emittance. These data are useful if the user wants to run the structure as a transfer line. It allows the program to transfer the beam matrix along the whole machine, in particular to calculate the beam matrix at the end of the structure, to display the evolution of the twiss parameters along the machine in **GR** etc. This section is not necessary for a run with a periodical structure, because the beam matrix is calculated by the periodical boundary conditions, even for a linear structure it can be omitted but in this case the analysis of the structure is very limited.
- mode : contains the keywords BEAM or BETA. Due to historical reasons these two keywords indicate if the data in the file describe a linear (BEAM) or a periodical structure (BETA). If this section is missing, the program assumes that the file describes a ring, consequently it calculates the twiss parameters with periodical boundary conditions. In case of a transfer line this section has to be included.
- list of elements : apart from the first line indicating the number of elements each line contains the description of one element, the details are described in option **MO**.
- structure : apart from the first line indicating the number of elements in the structure the following lines contain a sequence of elements 10 in a line to indicate the arrangement of the elements in the machine.
- particle type : indicates the particle either electron (E) or proton (P) used as beam in the machine.
- parameters of fit : contains the default parameters proposed for the input of the data for the options **AJ** or **AJ2**. The elements which are involved in the fit must be indicated by their numerical position in element list. If the elements are indicated by letters instead of numbers, BETA crashes.
- BUX/Z, T1X/Z : contains indications for the application of bumps and the first turn correction, more details are given in the description of option **MO2**.
- quad correctors : contains a list of quadrupolar correctors, more details are given in the description of option **MO2**.
- skew quad correctors : contains a list of skew quadrupolar correctors, more details are given in the description of option **MO2**.
- tables : contains a list of filenames which indicate the files which contain the tables for the description of mapped insertion devices (Appendix E).

It should be noted that BETA can read two types of different files, files which start with the list of elements following by the structure, and files which start with the indication of the beam matrix and with the mode section indicating the mode BEAM followed by the lists of elements. The latter feature has been included starting from version 3 of BETA . Furthermore, it allows the user to toggle between the modes transfer line and ring. It can be useful to intervene on a structure file, to modify it within a text editor. The other possibility is to take an already existing structure file and to modify it with the option **MO**.

¹ Note to adapt old beam files (extension .dat) the format of the beam matrix has to be changed. The new format of the beam matrix is 6 columns, (x, x', z, z', l, δ) instead of (x, x', δ , z, z').

6. DESCRIPTION OF THE DIFFERENT OPTIONS

The possible options available in BETA are listed below, while details on the input data for each option will be given in the following sections. An example of the results is available in Appendix B.

%%	end of run;
VI	use of the graphic editor;
Z	erase of the graphic memory;
X	show/print first analysis of structure
NLOG	display the BETALOG-file
\$	execute system command
AX	updating of axis for lattice functions plots and phase plane plots;
MO	modification of the actual lattice;
DE	definition of a new lattice;
FI	actions on the input file data (the structure file);
DIAG	plot of the tune diagram;
SY	plot of the schematic layout of the lattice;
GR	plot of the lattice functions;
TR	plot of the trajectory of an arbitrary particle along the ring;
CO	calculation of the closed orbit;
NUDP	calculation and plot of the variation of tune and lattice functions with momentum;
XXP	particle tracking in the horizontal phase plane (x, x');
ZZP	particle tracking in the vertical phase plane (z, z');
LDP	particle tracking in the longitudinal phase plane (l, δ);
XZ	calculation and plot of the dynamic aperture;
EXZ	plot of emittance blow-up;
DEFQ	generation of random errors in quadrupoles and analysis of the transfer matrix about the resulting closed orbit. Optional closed orbit correction;
DEFD	generation of random errors in dipoles and analysis of the transfer matrix about the resulting closed orbit. Optional closed orbit correction;
DEFM	generation of random errors in multipoles and analysis of the transfer matrix about the resulting closed orbit;
MO2	allows to introduce non periodic elements: dipole, quadrupole or skew quadrupole correctors, insertion devices. Their effect is analysed by running an error generating option (DEFD or DEFQ);
AL	calculation of the effect of vibrations;
AJ	linear lattice matching;
AJ2	second-order fits, including chromaticity adjustment, acceptance optimization, minimization of driving terms;
SX	plots of the driving terms of third-order resonances induced by sextupoles;
SXLD	sextupoles in the lattice are replaced by 3rd order straight multipoles;
LDSX	3rd order straight multipoles are replaced by sextupoles;
AVAR	sense of description of the lattice.

VI : GRAPHICAL EDITOR

This option calls a menu whose appearance depends on the graphics (Minigraph, GKS, Matlab) used. In case GKS or Matlab a new window appears which proposes different options, however, in case of Minigraph the menu appears as the other menus of BETA in the same window. Depending on the system the menu proposes the following options:

- Keep the actual graphics.
- Erase the actual graphics.
- Print the actual graphics, depending on the setup of the installation several printers are proposed. In case of Matlab the printout option is not proposed in the pop-up window. In Matlab this option can only be chosen by using the option File/Print.
- Store the actual graphics on file.

Z : ERASE GRAPHICS

shortcut to erase the actual graphics

X : ANALYSIS

shows the first analysis of the structure with the option to save the output without invoking the option **MO**. The saved output appears in the BETALOG file.

NLOG : REINITIALISE BETALOG FILE

removes the actual BETALOG file and opens a new one to take fresh data

\$: EXECUTE SYSTEM COMMAND

Beta only accepts one command line. After its execution it immediately toggles back into the BETA input mode.

AX : AXIS DEFINITION

The scales for all the drawings in phase spaces (x/x' , z/z' , l/δ) or in real space (s/x or z) can be redefined in this option. The units are :

x, z, s, l [m]

x', z' [m] for normalized coordinates, [mrad] for non-normalized coordinates

δ dimensionless

The standard input for each of the drawings are:

MIN VALUE ON HORIZONTAL AXIS	:	definition of drawing window
MAX VALUE ON HORIZONTAL AXIS	:	"
MIN VALUE ON VERTICAL AXIS	:	"
MAX VALUE ON VERTICAL AXIS	:	"

MO : MODIFY
DE : DEFINE

[FULL]

The option **MO** allows to determine in following options if the structure given is treated as a periodical (BETA mode) or a linear structure (BEAM mode). Furthermore by invoking **MO** additional settings applied in preceding options are removed i.e. **MO** resets BETA in its default state. However, its main task to allow modifications on the elements and their arrangement which constitute the machine.

For the latter part the various elements of the machine are displayed (list of elements), then the sequence of single elements whose order is the same as encountered in the superperiod is displayed (structure), and finally the number of superperiods can be modified. **MO** allows modifications of the elements and the structure, whereas **DE** erases all previous data and creates a new structure. Once the description is completed, BETA computes and displays the transfer matrix, twiss parameters, tunes of the ring.

During the calculation, the CTRL-C command can be used once to cancel the display of results, or twice to cancel the lattice analysis. If the number of quadrupoles or sextupoles in a superperiod is greater than 20, by default informations concerning them are suppressed. The qualifier FULL can then be used to get full information.

1. Type of machine :

Structure as RING(BETA) or TRANSFER LINE(BEAM)(RI/TL) : define if the following structure is taken as a periodical (BETA mode) or a single linear (BEAM mode) structure

INITIAL BEAM MATRIX(Y/N) : define if in BEAM mode initial conditions for the optical functions are given or not. If the read structure file is in mode BETA, in case of yes the beam matrix calculated from periodical boundary conditions is used. In case of no the following two options are skipped.

CHANGE INITIAL EMITTANCE(Y/N) : change the emittance as it is proposed by the default values. In case of a file in mode BETA and a zero vertical emittance the option proposes 1% of horizontal emittance as vertical emittance. Attention: If you skip this inquiry at the same step as the switch from RI to TL, the vertical emittance is set to zero. Hence you loose a part of the beam matrix information which cannot be recuperated afterwards (even if you set the vertical emittance to a non-zero value afterwards).

CHANGE INITIAL BEAM(N/Y) : possibility to change the proposed initial beam matrix elements by hand.

2. Single elements :

BETA displays the list of single elements, and then asks for editing commands allowing modifications of this list. The maximum number of different elements is 1200. The elements are identified either by their position in the element list or by an optional label (4 characters). When an element is inserted or deleted, all the following elements are renumbered and the structure list is subsequently modified.

parameters between square brackets [] are optional

"position" may be either the ordinal number of the element or its label

"type" is a keyword indicating the type of the element (see list below)

"list_of_parameters" is a list of 1 to 4 numerical parameters

"label" is 1 to 4 characters

The available commands are:

Insertion	I	position	[label]	type	list_of_parameters	
						<i>The new element is inserted after the specified element</i>
deletion	D	position				
modification	[CH]	position	[n.param	value	[n.param value ...]]	
						<i>If the parameter is omitted, the actual parameters are displayed and edited. Label is parameter 0</i>
next	<enter>					
						<i>displays the next page of single elements, or goes to structure input</i>
help	H					
new display	L					
print	P					
end	%%					

The available element types are :

Type	keyword	parameters			
Drift space	SD	length [m]			
Quadrupole	QP	length [m]	strength [m^{-2}] $K = B'_z / B\rho$		
Sector dipole	DI	angle θ [rad]	radius ρ [m]	field index n	sextupole component β
Edge focusing	CO	angle ε [rad]	unused	length L_{FF} [m] of fringing field	curvature [m] of pole face
Kick	KI	horizontal deflection [rad]	vertical deflection [rad]	dipolar corrector switch: 1:horizontal, 2:vertical, 3:both used only for C.O. correction	
Sextupole	SX	length [m]	strength [m^{-3}] $H = 1/2 \cdot B''_z / B\rho$		
Displacement of reference axis	DE	horizontal [m]	vertical [m]	momentum (position of the old origin in new reference system)	
Rotation of reference axis	RO	s-axis [rad]	x-axis [rad]	z-axis [rad] (angle of the new axis in the old reference system)	

Multipole (thin lens)	LD	strength [$m^{-N/2+1}$]	number of poles used only in option DEFM	r.m.s strength	
Skew multipole (thin lens)	LT	strength [$m^{-N/2+1}$]	number of poles	"	
R.F. cavity	CA	voltage V [V]	harmonic number h	kinetic energy E [eV]	
Septum	SE	x-coordinate of septum entry in phase space XS	x'-coordinate of septum entry in phase space XPS	bending radius [m]	length [m]
Time-dependent kick	KV	horizontal deflection [rad]	vertical (see next page) deflection [rad]		
Insertion device	ID	length [m]	max. bending radius [m]	$(k_x / k_y)^2$	period length [m]
pick-up monitor	PU	r.m.s. reading error (1st turn)	r.m.s. misalign. Error normal operation		
Mapped ID	IT	table number	number of iterations	energy scaling factor	
Linear sector	SQ	length in the 7 lines following the line with SQ the 7x7 matrix of the element			
Collimator	FE	hor. coll. size dX[m]	vert. coll. size dZ[m]		

Dipole (DI)

The field gradient is defined by:

$$n = -\frac{\rho}{B_z} \left(\frac{\partial B_z}{\partial x} \right)$$

and the sextupolar component by:

$$\beta = \frac{\rho^2}{2B_z} \left(\frac{\partial^2 B_z}{\partial x^2} \right)$$

Edge Focusing (CO)

This element must be used to take into account the focusing effect of the fringing field and the rotation of the pole faces (rectangular magnet). The correcting term resulting from spacial extent LFF of the fringing field is expressed as :

$$\psi = \frac{L_{FF} (1 + \sin^2 \epsilon)}{6\rho \cos \epsilon}$$

The [4,3] term of the pole face rotation matrix then becomes:

$$-\frac{\tan(\epsilon - \psi)}{\rho}$$

As entrance and exit pole faces are treated differently, the elements in the element list must be declared in the following order, without other data entry between them : entrance edge focusing, sector dipole, exit edge focusing.

Sextupole (SX)

To assure that transfers with sextupoles are symplectic, the length of sextupoles has to be very small, typically of 1e-6m. To get nevertheless the correct HI, the strength H (2. parameter) has to be scaled up by the same factor.

Multipole (LD/LT)

The integrated strength of a "2n-pole" multipole is given by :

$$\frac{1}{(n-1)!} \cdot \frac{\partial^{n-1} B_z}{\partial x^{n-1}} \cdot \frac{L}{B\rho}$$

Cavity (CA)

The [6,5] term of the transfer matrix is given by (where β is the relative velocity of the particle and R is the mean radius of the machine):

$$\frac{V}{\beta^2} \frac{h}{E R}$$

For tracking, a kick in energy is given to the particle, according to a sinusoidal RF field. CA is the only element which informs BETA about the energy of the beam. **Therefore a CA element must be put in the element list**, even if you don't put it in the structure list. Is it put in a structure list the voltage must be negative and divided by the number of superperiods.

Simulation of energy loss or acceleration is performed by adding a displacement in energy.

Time-dependent kicks(KV)

Up to 4 time-dependent kicks can be defined. For each kick, the change in horizontal and vertical deflection can be programmed over 20 turns and stored in a table during execution of the program. These data are entered by hand once the description of the structure is completed. When a tracking option is called, the amplitudes stored in the table are taken into account at each turn. After 20 turns, the kicks are set back to 0.

Septum(SE)

A septum can only usefully applied in the option **XXP**. Its only actual application is the study how many particles

will be intercepted by the septum during tracking of several turns. The septum has to be installed as very last element in the structure, otherwise this application does not work. More details in section **XXP** at p 27.

Insertion devices(ID)

The field expansion used in insertion devices [6] is:

$$B_x = (k_x / k_z)^2 \cdot B_0 \cdot \sinh k_x x \cdot \sinh k_z z \cdot \cos ks$$

$$B_z = B_0 \cdot \cosh k_x x \cdot \cosh k_z z \cdot \cos ks$$

$$B_s = -(k/k_z)^2 \cdot B_0 \cdot \cosh k_x x \cdot \sinh k_z z \cdot \sin ks$$

$$\text{where } k_x^2 + k_z^2 = \left(\frac{2\pi}{\lambda}\right)^2, \quad \lambda = \text{period length}$$

k_x expresses the x-dependence of the field; $k_x = 0$ for infinitely wide poles. For finite width, k_x is generally imaginary. The input values for BETA are : L = length, $\rho = (B\rho)/B_0$, $(k_x / k_y)^2$, λ

For all 1st order calculations, the insertion device is considered as a long focusing element with strengths :

$$f_x \text{ [m}^{-2}\text{]} = \frac{r^2}{2(1+r^2)\rho^2}, \quad f_y \text{ [m}^{-2}\text{]} = \frac{1}{2(1+r^2)\rho^2}, \quad \text{with } r^2 = (k_x / k_y)^2$$

For tracking, each period is divided into 10 slices, the particle is drifted to the middle of each slice, and is kicked by an angle corresponding to the field integrated along the thickness of the slice at its actual position.

Mapped insertion devices (IT)

These are insertion devices which are described by interpolation tables, one table for each plane, the tables provide a kick as a function of the coordinates of a particle passing the element. Further details are given in the appendix. Here only the three parameters to describe the element in the element list are explained.

- the number of the table : number of the line with the filename of the file containing the table in the structure subsection ***** TABLES *****
- the number of iterations : the number of intervals the element is splitted in to compensate for the thin lens approximation. One interval is often sufficient.
- a scaling factor to scale the energy : a possibility to vary the strength of the device. However, to vary the energy of the whole machine this factor should not be used.

The list of tables used in the structure has to be given at the end of the structure file. A subsection titled by ***** TABLES ***** has to be provided in the structure file. A line indicating the number of different tables follows the title. The lines following the indication of the number have each to contain a filename with complete path of a file containing the tables for both planes. The number of lines indicating filenames must correspond to the number of different mapped insertion devices used in the structure. Maximal 32 different tables are allowed.

Collimators (FE)

These are elements which can be used to trap particles whose horizontal or vertical excursion from the origin of the reference system is higher as a given value. They can be used in the tracking to prevent particles from keeping on moving at the place of the collimator if the absolute value of their coordinates exceed a given value. The jaws of the collimator can be visualized in the option GR. The action of the element is symmetric either the excursion of the particle is negative or positive.

Linear sector (SQ)

This type of element allows to describe an arbitrary element by its linear 7x7 element matrix. In the element list the matrix has immediately to follow the line specifying the element. Only one parameter on the line specifying the element is used, it indicates the length of the element. The element does not scale for an off-momentum particle. Furthermore, no parameter of the element can adjusted (**AJ(2)**) and it is not taken into account in option **AL**.

2. Structure input :

The superperiod is described by the list of single elements (defined by their rank in the single elements list or their label), as they appear along the circumference. The total number of elements in the superperiod is displayed, followed by a page of 200 elements (maximum per page). The maximum number of elements per superperiod is 3000 (with a limit of 2000 sextupoles+quadrupoles).

"position" means the element(s) or position after the element(s) defined by:

str.nu : element with ordinal number "str.nu" in the structure list
label : all the elements with name "label"
E/elem.nu : all the elements with ordinal number "elem.nu" in the element list
"range" represents all the elements between
ini.nu : position of the 1st element in the structure
[fin.nu] : position of the last element [optional, defaults to ini.nu]

The commands available to edit the list are :

Insertion	I	position	list_of_elements	<i>insertion after "position"</i>
deletion	D	range		
or	D	position		
modification	[CH]	position	list_of_elements	<i>replaces all the specified elements by "list_of_elements"</i>
move	M	position	range	<i>moves the specified range after "position"</i>
copy	C	position	range	<i>inserts a copy of the specified range after "position"</i>
next	<enter>			<i>displays the next page of elements, or goes to next input</i>
help	H			
list	L			
end	%%			

3. Number of superperiods

4. Particle type

Protons (P) or electrons (E) can be chosen. The default value is electrons.

5. Order of the transfer matrix

The default value is 1. In the most cases a first order transfer should be sufficient. Anyway, it is not a pure first order transfer, but a transfer in which element matrices of dipoles, dipole edges, quadrupoles, sextupoles, insertion devices and mapped insertion devices scale with $1/(1+\delta)$ if it is an off-momentum particle. However, element matrices of a kick and of an arbitrary element (SQ) do not follow the scaling. However, in a pure first order transfer the element matrices should not scale at all. Due to the fact that this is an academic case, the pure first order transfer is not applied in BETA. Additionally, in case of an off-momentum transfer of "first order" dipoles and dipole edges are not correctly treated with a linear 7x7 matrix, the second order calculation is used and the results are used to modify several first order matrix elements to become momentum dependent. Certainly, this is not a very clean method, however, it turned out to be the most efficient method to treat a dipole.

In case of a small machine this method can lead to a non-symplectic transfer.

Furthermore, in the tracking of "first order" sextupoles are treated as second order elements with scaling.

When the second order transfer is chosen, the transfer is carried out by transfer matrix enlarged by the second order coefficients. They are displayed within the first analysis of the structure if a second order transfer has been selected. The transfer has the following form:

$$\begin{pmatrix} x \\ x' \\ z \\ z' \\ l \\ \delta \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ x' \\ z \\ z' \\ l \\ \delta \\ 1 \\ x^2 \\ xx' \\ xz \\ xz' \\ xl \\ x\delta \\ x'x' \\ x'z \\ x'z' \\ x'l \\ x'\delta \\ z^2 \\ zz' \\ zl \\ z\delta \\ z'z' \\ z'l \\ z'\delta \\ l^2 \\ l\delta \\ \delta^2 \end{pmatrix}$$

The second-order matrix is given at the end of the first analysis of the structure (in fact the displayed matrix has to be "transposed" to correspond to the form of the matrix given here), however, contains some additional rows which are attributed to linear terms which are already covered by linear part of the transfer, i. e. the first 7 rows. And in fact the coefficients of these rows are always zero. However, in the transfer in BETA these coefficients are always taken into account without having any effect on the transfer. For this technical reason in BETA the second-order transfer is realised by a 7x35-matrix and not as a 7x28-matrix presented here.

In case of an off-momentum trajectory, in a second order transfer no scaling is applied. The dependency of the strength of some elements is contained in the second order coefficients, of course the power serie of δ is truncated after the second order. Therefore for off-momentum trajectories scaling can be more precise because

it takes the whole power serie of δ into account.

In the tracking with second-order transfer the user is asked if he wants an integral transfer by superperiod (MA) or a transfer element by element (EL). In the respect of today's available computing power the first choice should be abandoned in favor of the second. Only the latter method merits to be called a correct second-order transfer.

In fact, BETA uses 6 modes of tracking, indicated by an internal flag IRAP.

IRAP = 0 : tracking element by element, first or second order

1 : tracking with blocks, nonlinear elements are sandwiched by blocks of "linear elements"

2 : tracking in 2nd order "superperiod by superperiod" (MA), this is an obsolete mode

5 : tracking with blocks and treatment of errors, nonlinear elements are sandwiched by blocks of "linear elements"

3,4 : not installed

FI : FILES**FULL**

This is usually the first option the user applies: read data from an existing structure file, save data on a new or already existing file. A sub-menu proposes 5 options :

- 1 : saves the structure on a new file (or updates an old file)
- 2 : reads an existing file
- 3 : deletes an existing file
- 5 : list of "BETA" default directory
- 6 : exit

The default device name is "**DBETA:**", but any file specification may be given. The data are recorded on text files which can be edited or printed. After reading a file, the output is the same as for option **MO**, and the qualifier **FULL** has the same effect.

SY : LATTICE SYNOPTIC DRAWING

This option draws a schematic layout of the lattice, given the orientation of the machine with respect to the horizontal axis and the width of magnetic elements. This option is not working with Matlab graphics.

- INITIAL TETA : angle of the machine with respect to the horizontal axis [rad]
- DIPOLE WIDTH : dipole width [m]
- QUADRUPOLE WIDTH : quadrupole width [m]
- SEXTUPOLE WIDTH : sextupole width [m]
- DYNAMIC SCALES (Y,N) : if Y is entered, the scales are automatically calculated. On the other hand (N), the proposed scales can be modified:
- MIN VALUE ON HORIZONTAL AXIS :
- MAX VALUE ON HORIZONTAL AXIS : definition of drawing window
- MIN VALUE ON VERTICAL AXIS :
- MAX VALUE ON VERTICAL AXIS :

DIAG : TUNE DIAGRAM

This option sets the working point in the tune diagram and draws resonance lines $m v_x + n v_z = p$. The resonance type (systematic, random, both systematic and random, regular, skew) and the order, defined as $|m| + |n|$, can be selected. Once this is done, the program draws all these lines, provided they are located in the drawing window. Then, it is possible to superimpose on the graph predefined sets of working points; these data are recorded on text files which can be edited and modified.

NUX MIN , MAX : v_x range

NUZ MIN , MAX : v_z range

Starting from this drawing window, the program manages to obtain normalized scales.

OPTION (RE, TR, %%):

RE plot of resonance lines

TYPE OF RESONANCE (SY, RA, AL) : selects systematic resonances, random resonances, or both

REGULAR, SKEW, ALL (RE, SK, AL) : selects regular resonances, skew resonances, or both

ORDER : resonance order

MAX HARMONIC : maximum p value; all $m v_x + n v_z = p$ lines with $p < p_{max}$ will be drawn

TR plot of a given set of working points

FILENAME: a filename (without its extension) of a existing file or a new one has to be entered; its extension is supposed to be "nu". The file is displayed and can be edited, using an editor depending on the environment of the system.

GR : LATTICE FUNCTIONS PLOTS

For the two existing modes BEAM and BETA the option **GR** works slightly differently. In mode BEAM, the following options are available:

OPTION(ENV, PH, TR, EH, EV):

ENV plots the envelopes $\sqrt{\beta(s)}\mathcal{E}$ along the structure.

PH plots the horizontal and vertical phase advances along the structure. The phase advances are expressed in units of 2π ;

TR plots the trajectories along the structure as a result of first-order calculations (the effects of sextupoles are not taken into account);

EH plots the horizontal phase space ellipse at the end of the structure

EV plots the vertical phase space ellipse at the end of the structure

In all options of mode BEAM superpositions of different optics are possible. To that end the "plot another optics" has to be answered with yes. The user can load another optics from file or modify the already loaded optics with **MO** and at the recall of **GR** the same option is executed and the result superimposed on the first one.

Furthermore, in option TR superpositions of different trajectories are also possible. Answer yes at the inquiry "track another particle" and simply reenter a set of initial conditions and the resulting curves will be superimposed on the first ones.

In mode BETA the following options are available:

OPTION(BETA, PH, CO, ENV):

BETA plots the optical functions β_x, β_z and the dispersion η_x along the structure;

PH plots the horizontal and vertical phase advances along the structure. The phase advances are expressed in units of 2π ;

CO plots the closed orbit along the structure as a result of first-order calculations (the effects of sextupoles are not taken into account);

ENV plots the beam envelopes along the structure. It also computes the tune shifts induced by space charge effects. The following data are needed:

EX/PI (m.rad) : horizontal emittance

EZ/PI (m.rad) : vertical emittance

DELTAP/P MIN: minimum momentum deviation from the reference

DELTAP/P MAX: maximum momentum deviation from the reference

SPACE CHARGE CALCULATIONS (N/Y) :

If the answer is Y , the program asks for:

Q/A: charge over mass ratio

V/C: relative velocity of the particle

The following input data are then needed by all options (BEAM and BETA):

PRINT OUT OF THE VALUES (N/Y) :

The answer Y causes the plotted values to be written on a text file. Current filename (BETALOG.LOG) has not to be specified. The file can be edited or printed after the end of run.

AUTOMATIC SCALES (Y/N) : When the default value Y is entered, the scales of the plot are automatically calculated according to the current graph. For the "BETA" plot option, if the maximum dispersion is less than 5% of the maximum β , the dispersion function is multiplied by 10.

When N is entered , the proposed scales can be modified:

MULTIPLY DISPERSION BY 10 (N/Y) :

SMIN: minimum value on the horizontal axis

SMAX: maximum value on the horizontal axis

YMIN: minimum value on the vertical axis

YMAX: maximum value on the vertical axis

CO : CLOSED ORBIT

CO computes the closed orbit, taking into account the current alignment errors, for a given $\Delta p/p$. Depending on the choice of 1st or 2nd order made at the definition of the structure (FI, MO or DE), the method used is:

1st order: a particle is thrown at the position of the 1st order closed orbit, and is transferred through n turns around the machine using the 1st order tracking method : simple elements are cumulated in 1st order matrices, sextupoles are defined by their 2nd order matrix, multipoles are represented by a kick. The estimate for the closed orbit is taken as the average value of the successive positions of the particle and the process is repeated until the absolute deviation r^2 is smaller than a specified precision value. r^2 is defined as:

$$r^2 = \sum_{k=1}^{k_{\max}} (y_f - y_i)^2$$

y represents the particle coordinates x, x', z, z', l or δ . The summation is made over the 6 coordinates ($k_{\max} = 6$) if a cavity is present in the structure. Otherwise $k_{\max} = 4$.

2nd order : a particle is thrown at the position of the 1st order closed orbit, and is transferred around the machine using either one 2nd order matrix per element, or one cumulated 2nd order matrix per superperiod. Estimate of the closed orbit and convergence criteria are the same as for 1st order.

DP/P : relative momentum deviation of the particle

PRECISION C.O. : absolute precision of the closed orbit determination

NB STEPS, NB TURNS/STEP : maximum number of iterations and number of turns at each iteration for closed orbit determination

For 2nd order only:

TRANSFER PER ELEMENT (EL), SUPERPERIOD (MA):

EL one 2nd order matrix per element

MA one 2nd order matrix per superperiod

The 6 coordinates of the closed orbit are displayed at each iteration. The process can be interrupted by CTRL-C command; it may be restarted or definitively stopped according to the answer to the question:

CONTINUE ? (Y,N) :

If convergence is achieved, the closed orbit is then drawn. If the axis for transport have not been defined yet, the program asks for them now (see AX).

NUDP : VARIATION OF TUNES WITH MOMENTUM

NUDP computes the variation of the lattice functions (β, α , tunes) with momentum. This is achieved by finding the chromatic closed orbit (1st and 2nd order) for a user-defined range of momentum. The optical parameters are then obtained by analysing the transfer matrix computed about this closed orbit in the following way :

- quadrupole strengths are scaled by a factor $1/(1 + \Delta p/p)$;
- sextupoles are replaced by thin quadrupoles and skew quadrupoles according to the closed orbit position and the scaling factor $1/(1 + \Delta p/p)$;
- dipole and edge focusing matrices are modified, using the chromatic terms of the 2nd order matrix and the momentum deviation;
- multipoles, cavities, displacements and rotations are not allowed.

The parameters α_x, β_x, ν_x and α_z, β_z, ν_z are stored for each value of $\Delta p/p$ and corresponding curves can then be drawn.

PRECISION C.O. : absolute precision of the closed orbit determination
NB STEPS, NB TURNS/STEP : maximum number of iterations and number of turns at each iteration for closed orbit determination
DP/P MIN : minimum value of $\Delta p/p$
DP/P MAX : maximum value of $\Delta p/p$
DP/P STEP : step

For each value of $\Delta p/p$, the 6 coordinates of the closed orbit are displayed. The process can be interrupted by CTRL-C; it may be restarted or definitively stopped according to the answer to the question:

CONTINUE ? (Y,N) :

Once the values are stored, the different functions can be drawn.

PLOT (%%,NUX,BX,AX,NUZ,BZ,AZ) :

NUX	fractional part of the horizontal tune
BX	horizontal β
AX	horizontal α
NUZ	fractional part of the vertical tune
BZ	vertical β
AZ	vertical α
%%	exit

For each option, the program asks for the vertical scale (default values are the previous values for the same option), draws the curve and asks for the next plot.

XXP :
ZZP : **TRACKING**
LDP :

This option tracks particles, draws the phase space plot of the particle turn after turn, and analyses its position to obtain its frequency spectrum. **XXP** plots and analyses the horizontal position, **ZZP** the vertical position, **LDP** the longitudinal position. According to the choice of the tracking option (**XXP**, **ZZP** or **LDP**), the current phase space plot is displayed. The coordinates of the tracked particle can be stored over 5000 turns; therefore any phase space plot or frequency spectrum can be redrawn, once tracking is completed. Multipole field errors, misalignment errors, insertion devices or synchrotron oscillations (if there is a cavity in the structure) are taken into account during tracking.

Once given the momentum of the particle, BETA computes the chromatic closed orbit associated with this momentum and then asks for the betatron coordinates (coordinates with respect to the closed orbit position). The Fourier analysis uses these coordinates, but the phase-space plot represents the position with respect to the reference system (section 2.1 figure 1). Depending on the choice of 1st or 2nd order made at the definition of the structure (**FI**, **MO** or **DE**), the method used for tracking differs:

2nd order : transfer through 2nd order matrices uses either one matrix per element or one cumulated matrix per superperiod. Tracking element by element has to be preferred to the tracking superperiod by superperiod, the latter method can give unacceptable truncation of the transfer. In both cases tracking with 2nd order matrices for dipoles can lead to non-symplectic transfer.

1st order : all the sequences of simple elements (drift spaces, quadrupoles, dipoles with edge focusing², kicks, change of reference system) are represented by a cumulated 1st order matrix. A sextupole is represented by its 2nd order matrix, and multipoles by a kick. In cavities, a kick in energy according to a sinusoidal RF field is given. In insertion devices each period is divided into 10 slices where a kick according to a perfect undulator field is given. Misalignment errors previously introduced with the options **DEFD** or **DEFQ** are taken into account.

Once the definition of particle coordinates is completed, the energy dependent single elements matrices are rescaled. The same procedure is applied after each energy variation (cavity or displacement in energy).

Different criteria can be used to interrupt the tracking. The user has the choice between no criterium except numerical overflow (the particle has gone to infinity), an absolute blow-up criterium (the user enters a certain emittance value which the particle is not allowed to exceed), or a relative blow-up criterium which stops the tracking if the actual particle emittance in units of the initial emittance exceeds a user provided value.

1. closed orbit

DP/P : relative momentum deviation of the particle
PRECISION C.O. : absolute precision of the closed orbit determination
NB STEPS, NB TURNS/STEP : maximum number of iterations and number of turns at each iteration for closed orbit determination

For 2nd order only:

TRANSFER PER ELEMENT (EL), SUPERPERIOD (MA):

EL : one 2nd order matrix per element
MA : one 2nd order matrix per superperiod

² in case of an off-momentum trajectory, the element matrix of a dipole and a dipole edge is only of "quasi first-order" (see p.15)

The 6 coordinates of the closed orbit are displayed.

2. coordinates of the particle

OPTION (%%,EP,YYP,NP,DE) :

YYP the particle is defined by its 6 coordinates x, x', z, z', l, δ

BETA proposes as default values either the initial conditions (choice IN) or the final position (choice FI) of the last tracked particle.

EP the particle is defined by $EX/PI = [x^2 + (\alpha x + \beta x')^2] / \beta$

$$PHIX = \arctan [(\alpha x + \beta x') / x]$$

and equivalent in vertical and longitudinal directions. As in YYS, the default values can be the initial or final positions of the last tracked particle.

NP a set of NX,NZ,NL particles with emittance EX/PI,EZ/PI,EL/PI and phases equally distributed in $[0, 2\pi]$ is generated in the horizontal, vertical and longitudinal phase spaces. Default values for ALPHA and BETA of the tracked beam are values of the adapted beam.

DE closed orbit coordinates. The displayed values have been computed in the first step of the option and should not be modified.

%% definition of the particle completed.

3. tracking parameters

BLOWUP TEST: NO: no blowup test carried out.

A: absolute blowup test carried out. The user has additionally to enter the coordinates of a reference particle which will be used to calculate a reference emittance. If during the tracking of a particle the value of the reference emittance is exceeded the tracking will be interrupted.

R: relative blowup test carried out. Enter a blowup ratio. If the ratio given by $(\mathcal{E}_{turn n} / \mathcal{E}_{initial})$ exceeds the for the option prompted blowup ratio the tracking will be interrupted. $\mathcal{E}_{initial}$ is defined as the initial betatron emittance of the tracked particle and $\mathcal{E}_{turn n}$ as the betatron emittance of the particle at turn n.

NUMBER OF TURNS : number of turns for tracking

MINIMUM NU : frequency range for Fourier analysis (given in fractional part of the tune)

MAXIMUM NU :

NB OF CHANNELS : the default value which is proposed corresponds to the best achievable resolution, according to the number of turns and the frequency range

NORMALIZED, NON-NORMALIZED VERTICAL SCALE(NO,NN) :

The spectral amplitude of the different peaks can be either normalized to 1 or not.

EMERGENCY EXIT: NO continue, %% provokes fast exit, AX changes the axes without NO CHANGE, CHANGE AXIS, TOTAL EXIT: quitting the option

If the axis for phase space plot have not been defined yet, the program asks for them now (see **AX**)

PHASE SPACE... (NO,NN,CI) :

The coordinate system used for the plot can be:

NO	horizontal axis	x	[m]
	vertical axis	$\alpha x + \beta x'$	[m]
NN	horizontal axis	x	[m]
	vertical axis	x'	[rad]

Tracking may be stopped by a CTRL-C command; it may be restarted or definitively stopped according to the answer to the question:

CONTINUE ? (Y,N) :

The following options are available at the end of tracking:

EXIT (%%), SPECTRUM(S), NEW TRACKING (T), NEW DISPLAY(D) :

Draws on the screen the frequency spectrum of the particle for the chosen phase space, starts a new tracking, or draws another graph. In that case, the frequency range has to be specified if a frequency spectrum is plotted; if the axis for phase space plot have not been defined yet, the program asks for them now (see **AX**).

If a septum is installed as the last element of the structure the display of the horizontal phase space plot shows the position of the septum shield. The tracking is interrupted as soon as a particle has a larger amplitude than the septum shield. The coordinates of the intercepted particle are displayed. In case of option NP, a whole ensemble of particles can be tracked what allows to study how many particles will survive the passage at the septum.

XZ : DYNAMIC APERTURE

The option XZ uses the tracking method described in option XXP to draw the limit of the dynamic aperture: particles are sent with derivatives $x' = z' = 0$, x and z are incremented according to a predefined grid, and checked for a given number of turns. Initial positions are represented by an amplitude $A = A_i + n\Delta A$ and angle $\phi = \phi_i + p \cdot \Delta\phi$ such that $x = -A \cos(\phi)$ and $z = A \sin(\phi)$.

For each angle ϕ , the stable particle with the maximum amplitude is plotted as the limit of the dynamic aperture. The following effects may be taken into account: multipole field errors, fixed displaced energy (by specifying a Dp/p value) or synchrotron oscillations (if there is a cavity in the structure), insertion devices. Misalignment errors previously introduced with the options DEFD or DEFQ are taken into account.

Different criteria can be used to interrupt the tracking. The user has the choice between no criterium except numerical overflow (the particle has gone to infinity), an absolute blow-up criterium (the user enters a certain emittance value which the particle is not allowed to exceed), or a relative blow-up criterium which stops the tracking if the actual particle emittance in units of the initial emittance exceeds a user provided value.

1. X-Z scan

MINIMUM ANGLE (deg) : ϕ_i starting point [°]
MAXIMUM ANGLE (deg) : ϕ_f final point [°]
ANGLE (deg) STEP : step $\Delta\phi$ (positive or negative) [°]

Default values correspond to 40 angular steps.

MINIMUM AMPLITUDE : A_i starting point [m]
MAXIMUM AMPLITUDE : A_f final point [m]
AMPLITUDE STEP : step ΔA (positive only) [m]

2. closed orbit

DP/P : relative momentum deviation of the particle
PRECISION C.O. : absolute precision of the closed orbit determination
NB STEPS, NB TURNS/STEP : maximum number of iterations and number of turns at each iteration for closed orbit determination

For 2nd order only:

TRANSFER PER ELEMENT (EL), SUPERPERIOD (MA):

EL one 2nd order matrix per element
MA one 2nd order matrix per superperiod

The 6 coordinates of the closed orbit are displayed.

3. tracking parameters

X REF : X coordinates of a reference particle for the emittance test
Z REF : Z coordinates of a reference particle for the emittance test
NUMBER OF TURNS : number of turns for tracking
BLOWUP TEST: : NO: no blow up test carried out.
: A: absolute blowup test carried out. The user has additionally to enter the coordinates of a reference particle which will be used to calculate a reference emittance. If during the tracking of a particle the value of the reference emittance is exceeded the tracking will be interrupted.
: R: relative blowup test carried out. Enter a blowup ratio. If the ratio given by $(\mathcal{E}_{turn\ n} / \mathcal{E}_{initial})$ exceeds the for the option prompted blowup ratio the tracking will be interrupted. $\mathcal{E}_{initial}$ is defined as the initial betatron emittance of the tracked particle and $\mathcal{E}_{turn\ n}$ as the betatron emittance of the particle at turn n.
LINE STYLE : straight lines or dotted lines are available

4. scales

AUTOMATIC SCALES (Y / N) : allows overlay of 2 graphs if « yes » is entered at a second run of XZ.

Y the drawing window is 10% wider than the grid defined for scanning x and z

N user-defined window; the following data is needed:

XMIN :]
XMAX :] horizontal and vertical scales
ZMIN :]
ZMAX :]

The program displays the particle coordinates and draws the dynamic aperture with respect to the closed orbit. Results are printed on the current log file. Execution may be stopped by a CTRL-C command; it may be restarted or definitively stopped according to the answer to the question:

CONTINUE ? (Y,N) :

EXZ : EMITTANCE BLOW-UP

This option computes turn after turn the evolution of the vertical and horizontal emittances of a set of particles. In the first mode, this evolution is drawn on the screen, and several particles can be superimposed on the graph. In the other mode two parameters (quadrupole strengths for instance) are varied, for each point v_x, v_z and the maximum emittance blow-up are computed for a given number of turns, and contour-lines of the surface "blow-up as a function of v_x, v_z " are plotted.

The emittances taken into account are computed with respect to a reference particle :

$$(x, x', z, z', l, \delta) \quad \text{is the test particle}$$
$$(x_{ref}, x'_{ref}, z_{ref}, z'_{ref}, l_{ref}, \delta_{ref}) \quad \text{is the reference particle}$$

The two particles are tracked and the emittance is given by:

$$\varepsilon = \gamma \cdot (y - y_{ref})^2 + 2\alpha \cdot (y - y_{ref}) \cdot (y' - y'_{ref}) + \beta \cdot (y' - y'_{ref})^2$$

where y stands for x or z .

1. graphic options

NORMALIZED, NON NORMALIZED EMITTANCES (NO,NN) :

NO the plotted variable is $\varepsilon / \varepsilon_{initial}$

NN the variable is ε

If $\varepsilon_{initial}$ (ε_{x0} or ε_{z0}) is equal to 0, normalization is performed by the non-zero emittance (ε_{x0} or ε_{z0})

PLOT EX and EZ, EX, EZ (XZ,X,Z) :

X ε_x as a function of the number of turns is plotted

Z ε_z as a function of the number of turns is plotted

XZ ε_x and ε_z are superimposed

R/R0 MIN,MAX : scale for the vertical axis of the graph. According to the choice of normalized or non-normalized emittances, the values are in mrad or dimensionless.

2. coordinates of the particles

The absolute coordinates of the reference particle are given first. The proposed default values are either the initial coordinates (choice IN) or the final coordinates (end of previous tracking, choice FI) of the last reference particle.

The coordinates of the test particle with respect to the reference particle are then entered :

OPTION (%%,EP,YYP,NP,DE) :

YYP the particle is defined by its 6 coordinates x, x', z, z', l, δ with respect to the reference. BETA proposes as default values either the initial conditions (choice IN) or the final position (choice FI) of the last tracked particle.

EP the particle is defined by $EX/PI = [x^2 + (\alpha x + \beta x')^2]/\beta$
 $PHIX = \arctan [(\alpha x + \beta x')/x]$

and equivalent in vertical and longitudinal directions. As in YYP, the default values can be the initial or final positions of the last tracked particle.

NP a set of NX,NZ,NL particles with emittance EX/PI,EZ/PI,EL/PI and phases equally distributed in $[0, 2\pi]$ is generated in the horizontal, vertical and longitudinal phase spaces. Default values for ALPHA and BETA of the tracked beam are values of the adapted beam.

DE meaningless in this option.

%% definition of the particle completed.

BLOWUP TEST: NO: no blow up test carried out.

A: absolute blowup test carried out. The user has additionally to enter the coordinates of a reference particle which will be used to calculate a reference emittance. If during the tracking of a particle the value of the reference emittance is exceeded the tracking will be interrupted.

R: relative blowup test carried out. Enter a blowup ratio. If the ratio given by $(\mathcal{E}_{turn\ n} / \mathcal{E}_{initial})$ exceeds the for the option prompted blowup ratio the tracking will be interrupted. $\mathcal{E}_{initial}$ is defined as the initial betatron emittance of the tracked particle and $\mathcal{E}_{turn\ n}$ as the betatron emittance of the particle at turn n.

NUMBER OF TURNS : number of turns for tracking

3. mode

VARIABLE PARAMETERS (N/Y) :

N the current structure is kept

Y two variable parameters can be defined

NUMBER OF VARIABLES : 1 or 2 variables are allowed. For each variable, the following parameters are necessary :

ELEMENT NB : number of the element to be varied

PARAMETER NB : number of the variable parameter for the given element

MIN,MAX,STEP : lower and upper limits of the parameter and step size

For each value of the variable parameters, the tunes are computed. The test particle and the reference particle are tracked, the evolution of the emittances is drawn and the maximum blow-up is stored.

OPTION (%%,AX,ML,GE,VI,Z,INT) : when the scan is completed, several options allow to control the output of the contour-lines :

AX axis definition, the tune variation, the blow up variation etc.

ML draws the lines corresponding to constant values of the variable parameters

GE draws the contour-lines

VI graphic editor

Z clear screen

INT has become obsolete

%% exit

MO2 : MODIFICATION OF CORRECTOR SETTINGS

This option provides an overview and the possibility of modification of special elements which does not appear in every superperiod, in particular all types of correctors and insertion devices.

The quadrupolar corrector positions have to be given by a subsection of the structure file. The subsections are titled by the lines

*** QUADRUPOLE CORRECTORS *** : quad(rupolar) correctors
*** SKEW QUAD CORRECTORS *** : turned quadrupolar (skew quad) correctors

For the two types of quad correctors the subsection of the structure file should be provided as following:

The line following the title has to contain 2 figures, the first one indicates the number of correctors in a corrector setup, the second figure indicates how many times the given corrector setup reappears along the machine circumference. It is a kind of repetition rate. The period in terms superperiods of the corrector setup is then the total number of superperiods divided by the repetition rate (If the setup appears only once a 1 should be used as 2.figure). The next lines represent a listing of positions of each corrector given by its position within the superperiod (NS) and the number of superperiod (NM). The number of these lines should be equal to the first figure given in the very first line. Note if you change the machine to developed machine (all elements in one superperiod) these figures have to reset correctly.

The correctors have to be installed depending on their type in existing quadrupoles or kickers.

- quadrupolar correctors in existing quadrupoles
- skew quad correctors in existing kickers

The concerned quadrupoles have to have a non-zero gradient integrals, because the values of the quad correctors are relative values which refer to the absolute strength of the quadrupole in which they are installed.

Corrector dipoles must be described in the element list as well in the listing of the structure as "Kick" elements (code **KI**) with the third parameter indicating how the element is to be used:

0	no correction with this element
1	horizontal correction
2	vertical correction
3	horizontal + vertical correction

If the kickers are used for some dedicated correction method, the following subsections have to be added in the structure file:

T1X or *T1Z* : 1st turn correction in horizontal or vertical plane

Each line of this section contains 3 integer numbers, corresponding to one machine section

- position of the corrector in the structure list
- position of the last beam position monitor in the structure list
- 0 (this zero is required)

All the sections of the first superperiod of the machine are described successively, BETA assumes that the corrector and bump arrangement is identical in the following superperiods. To describe bumps extending into the 2nd superperiod one uses element numbers incremented by the total number of elements in one superperiod. At the end of the first turn, the first corrector is readjusted to close the 1st turn both in position and angle, assuming that the correction of the injected beam misalignment is performed with other unknown elements.

BUX or *BUZ*: local bumps in horizontal or vertical plane Each line of this section contains 3 integer numbers, giving the position in the structure list of the 3 correctors producing one local bump. All the bumps belonging to the first superperiod of the machine have to be described. To describe bumps extending into the 2nd superperiod one uses element numbers incremented by the total number of

elements in one superperiod.

If **MO2** is invoked the following menu appears:

ELEMENT TYPE (KIX,KIZ,QP,SK,ID): choose the element type

KIX	horizontal kicker
KIZ	vertical kicker
QP	quadrupolar corrector
SK	turned quadrupolar (skew quad) corrector
ID	insertion device

It follows an overview of the installation of the chosen type of element in the whole machine (only the first 6 superperiods are displayed, the others are hidden). The user has then the following possibilities:

%%	quit the option
ns nm	set specific corrector by entering its position in superperiod, the superperiod and its value
ALL	give all correctors the same value
RAND	give random value to all installed correctors (of the same type)
HARM	performs a harmonic analysis either of the orbit or of the given kicker setup (only works for kickers)

To see the corrector values of the hidden part of the table use "ns 'superperiod number of hidden part' 0" (Use the last trick only if you are sure that the corrector at that position has the value 0). Note also: All kicker and corrector values are reset to 0 if the option **MO** is invoked.

DEFQ : GENERATION OF RANDOM QUADRUPOLE ERRORS [SAVE REC FILE]

In this option transverse or longitudinal random displacements, rotation errors and integrated gradient errors in each quadrupole can be applied to the elements of the structure. Machines affected in this way are calculated and analysed. Correction methods can be applied to correct the effects created by the errors.

The program performs the following analysis:

- It calculates of the closed orbit by tracking. The new transfer matrix is computed along the closed orbit.
- If a correction of the errors is desired the corrector arrangement has to be described in subsections of the structure file. Their format is described in the option **MO2**.
- Optional correction of the closed orbit. perturbed tunes , β -values, emittances...
- The dynamic aperture (defined by 5 points and its surface) may be computed for each machine.
- Statistics on various parameters of the perturbed machines is computed and displayed at the end of the run. A test on the maximum closed orbit deviation allows to reject from this statistics unwanted machines.

The errors can be generated in three ways:

- The wanted type of error is randomly generated from an initial random seed number which can be changed by hand at each new run. All elements affected by the indicated type of error are concerned. Each new seed number leads to a new error constellation of the machine elements. No control of the error of an individual element is given. However, the rms of the error strength is required from the user.
- The qualifier RECOVER allows to read a file which contains already saved rms values of error strengths and corrector values and correction steps of a run carried out prior to now issued one. This file must have been

created before in a run by applying the qualifier SAVE to DEFQ. Some of the parameters stored in the file can be changed as the number of sample machine to be studied as well as new correction steps can be added.

- Applying the qualifier FILE a file containing a list of individual errors affected to each element of the machine is read. The file has to be created exterior of BETA by a text editor. This option allows to study the behaviour of a machine under a precisely known set of errors contrary to the first method.

Once a set of errors has been generated, the tracking takes into account the effects of these errors. Errors introduced in the different **DEFQ**, **DEFD** and **DEFM** options are cumulated. To cancel a previously generated set of errors, one has to use the **MO** command or to set the standard deviations to 0 in all the 3 options. During calculation, a CTRL-C command can be used to stop the process.

Closed orbit correction :

BETA can use 5 methods for correcting closed orbit distortions :

- first turn steering with (**T1**) or without sextupoles (**TS0**): a beam is injected on axis (optionally with random position and angle errors) at the beginning of the machine description. The first turn is divided in sections (from one corrector to a position monitor) described in the structure file. The beam is steered in each section (powering the corrector to minimise the displacements on the different monitors of the section) as in a transfer line.
- most effective corrector method **EF**: this method uses one corrector in each plane at each iteration.
- local bump method **BU**: the definition of the various bumps to be used (3 correctors/bump) must be given in the structure file. Bump values are computed in order to minimise the sum of the squares of beam displacements seen on all the position monitors located in the bump range. Each iteration computes new values for all the bumps.
- Single value decomposition **SVD**: Now widely used method to correct the closed orbit in very efficient way. The user has to provide the number of eigenvectors he wants to apply in each plane for the correction.

For all methods, the uncertainty of the monitor measurement and the corrector settings can be taken into account. Limitations on corrector strengths in each plane can be given.

Pick-up monitors are indicated by elements with code **PU**. The first parameter gives the rms value of the random error introduced when reading the beam position during the 1st turn steering, the 2nd parameter gives the rms-value used for the other methods.

Resonance correction :

In case of gradient or rotation errors of the quadrupoles a correction of the occurring second order resonances can be applied (Of course this option can also be used without application of a correction). BETA carries out a harmonic analysis of the excited resonances. Two types of corrections can be applied:

- Correction by quad correctors. In the correction menu this method can be invoked by typing **CQ**.
- Correction by skew quad correctors. In the correction menu this method can be invoked by typing **CS**.

A subsection of the structure file allows to define these correctors. The option **MO2** allows the control of these correctors. A sufficient number of correctors should be installed. The program will apply a least-square fit to adapt the corrector values at best to cancel the excitations.

1.generation of errors

If the qualifier REC is not used, parameters for generating random errors are given here:

SIGMA X : transverse displacement standard deviation [m]

SIGMA Z : vertical displacement standard deviation [m]

SIGMA DG/G : integral of gradient
SIGMA S : azimuthal displacement standard deviation [m]
SIGMA ROT S : rotation around the s axis
ACTUAL SEED NUMBER : an arbitrary value resulting from the previous run is proposed.

If the qualifier RECOVER has been used, the 4-character file identifier is requested:

RECOVER FILE I.D. (4 characters):

If the qualifier SAVE has been used, the 4-character file identifier is requested. If both qualifiers are used, with the same id., a new version of the file is created for saving the new values.

SAVE FILE I.D. (4 characters) :

NUMBER OF SAMPLES : number of different sets of errors

The user has to assure that in case of SAVE the program has the permission to write the file to the DBETA-directory. The file will appear under the name structurefilename.STATxxxx in the DBETA-directory. It is an unformatted file.

If the qualifier FILE has been applied the user has to put at least one of the sigma's of different error types to nonzero which are followed by the prompt: *ERROR FILE ID. (4 characters)*: Here the user enters the last 4 characters of the error filename; the latter has to follow a strict naming convention: structurefilename.ERRxxxx of the actual structure. The file has to reside in the actual DBETA-directory. The file has to be set up as follows:

The first line of the file contains 3 bitmasks to indicate the figures given. The first bitmask is for dipoles, the second for quadrupoles and the third for the multipoles. If only one type of element is used the bitmasks of the others has to be set to zero. The parameters of the individual elements are given in the following lines, the type of the element has to be placed at the beginning of each line (DI, QP, SX, LD, LT, RO). The order of the elements has strictly to follow the order of these elements in the structure. For a multi/quadru/dipole up to 5/5/10 parameters can be given following the the order used in DEF/Q/D. If for instance only the 3th and the 5th parameter have to be read the bitmask has to be set to $2^{**}(3-1)+2^{**}(5-1)$, in case of all parameters $2^{**}(1-1)+...+2^{**}(5-1)$ etc. Additionally some precautions have to be applied: Errors of strengths can only be applied on LD's and LT's. To apply a strength error on a sextupole it has to be transformed into a thin lens LD. If a run with different types of elements (DI and QP (LD) for instance) is wanted DEFQ FILE as well as DEFQ FILE (or DEFM FILE) has to be issued. Finally in all options (except DEFM) at least one of the RMS-values prompted for has to be put non-zero (Of course in this case its numerical value is not taken into account).

2. closed orbit

DP/P : relative momentum deviation of the particle

PRECISION C.O : absolute precision of the closed orbit determination

NB STEPS, NB TURNS/STEP : maximum number of iterations and number of turns at each iteration for closed orbit determination

3. closed orbit correction

CLOSED ORBIT CORRECTION (N, Y) : if a closed orbit correction has to be computed, additional parameters are required :

max. horizontal correction : maximum strength of horizontal correctors (rad)

max. vertical correction : maximum strength of vertical correctors (rad)

corrector setting error (rad) :

initial <x > : r.m.s for random position error of the injected beam (used only in the 1st turn correction method)

initial <x'> : r.m.s for random angle error of the injected beam (used only in the 1st turn correction method)

If several sample machines are to be computed, the correction procedure is defined here. Otherwise, correction is performed step by step under the user's control.

CORRECTION PROCEDURE : the list of correction steps is defined by codes:
T1 "1st turn" method
nEF n iterations of "most effective corrector" method
nBU n iterations of "local bump" method
nSVDH/V n iterations of "SVD horizontal" res. "SVD vertical" method

For example, the default value T1 EF 6BU means 1st turn correction, followed by one iteration of "most effective corrector" method and the 6 iterations of "local bump" method.

4. second order resonances

This option is only useful if gradient errors or rotational errors exist in the machine. If the user invokes this option by typing "Y", he will be asked for which resonances we want excitation levels to be calculated.

2*NUX harmonic $\pm \delta/2$ $\delta \leq 65$: central resonance line and the total number of lines calculated
 2*NUZ harmonic $\pm \delta/2$ $\delta \leq 59$: " " " " " "
 NUX-NUZ harmonic $\pm \delta/2$ $\delta \leq 53$: " " " " " "

If the user wants no line, enter 0 as 2.figure, only the central line, enter 1. Otherwise enter higher numbers. The amplitudes of the excitations will be calculated and listed on the screen, they can be visualized by using the options HCQ and HSK.

5. dynamic aperture

DYNAMIC APERTURE (N,Y) : if the dynamic aperture has to be computed, additional parameters are required :

X- LIMIT : limit of the search in the $x < 0$ direction
X- STEP : step in the $x < 0$ direction
X+ LIMIT : limit of the search in the $x > 0$ direction
X+ STEP : step in the $x > 0$ direction
Z+ LIMIT : limit of the search in the $z > 0$ direction
Z+ STEP : step in the $z > 0$ direction
NUMBER OF TURNS : number of turns for tracking
REFERENCE X VALUE : X coordinates of a reference particle for the emittance test
REFERENCE Z VALUE : Z coordinates of a reference particle for the emittance test
BLOW-UP : maximum emittance ratio ($\epsilon / \epsilon_{ref}$) allowed during tracking
 (see option XZ for the 3 last parameters)

6. graphics and statistics

When several sample machines are computed, statistics are built for closed orbit position at pick-up locations (**RMS PU**), closed orbit position at the beginning of each superperiod (**RMS CO**), maximum closed orbit anywhere around the machine (**MAX CO**), tunes, emittance projections and cross sections, coupling, orbit in the

sextupoles, betafunctions, dispersion and the beam sizes in both planes. The threshold given here on the maximum closed orbit reached all around each machine (in both planes) after the last correction step allows to reject some machines from the statistics on the last step.

Max c.o. limit for statistics : The default value (1 m) is supposed to accept all machines.

If only one set of errors is run, one gets control after each correction step:

If closed orbit correction has been selected, BETA asks for the next correction step:

NEXT STEP (%%,SVD,T1,TS0,EF,BU,S0,CQ,CS,MO2,GR) :

- %% : default, quits completely DEFQ. Be careful to use it.
- SVD : apply single value decomposition to correct the closed orbit
- T1 : first turn correction with sextupoles
- TS0 : first turn without sextupoles
- EF : most efficient correction
- BU : correction using bumps
- S0 : switch the sextupoles on
- CQ : correction of quadupolar resonances
- CS : correction of 2nd order skew resonances
- MO2 : call option **MO2** to change corrector settings
- GR : call plot options

If the corresponding correctors are not installed, the applied correction has no effect.

PLOT (%%,OTF,COX,COZ,BETX,BETZ,DX,DZ,ENVX,ENVZ,ENVS,CPL,HARM,HCQ,HSK,HH,VI) :

- OTF : writes the CO-coordinates (x,z) at the pickups to file. Only 4 characters are required. The file is saved under the name "structurefilename.COxxxx" in the DBETA-directory. However, the user has to assure the write permission to that directory.
- COX (COZ) : horizontal (vertical) closed orbit
- BETX (BETZ) : horizontal (vertical) beta values
- DX (DZ) : horizontal (vertical) dispersion functions
- ENVX (ENVZ,ENVS) : horizontal (vertical,longitudinal) envelopes (using for electron machines the equilibrium emittances of the perturbed machine)
- CPL : plot of the coupling obtained by the emittances by projection (see 2. and 3. section of Appendix D)
- HARM : harmonic analysis of the closed orbit position on the pick-ups
- HCQ : shows the excited 2nd order resonances in horizontal and vertical plane, only active if *2nd ORDER RESONANCES* has been activated.
- HSK : show the excited 2nd order skew resonances, only active if *2nd ORDER RESONANCES* has been activated.
- HH : shows for every calculated resonance (if *2nd ORDER RESONANCES* has been activated) its corresponding vector in a two-dimensional plot. The vectors of the correctors and the sum of all correction vectors are also shown. However, the enumeration of the resonances is different as usual. To get the plot of a calculated resonance enter the number which is displayed before as HHindex preceding the corresponding resonance in the display of the result of the calculation of the excited resonances.

VI : calls option **VI**
FIRST, LAST CELL : selects the cells over which the selected function is drawn. The values along the different cells are superimposed
AUTOMATIC SCALE (Y,N) :

If several sets of errors are run, the graphic option selected at the beginning remains in effect for the whole run, and different sample machines are superimposed:

PLOT (%%,OTF,COX,COZ,...) : |
FIRST, LAST CELL : | same function as in previous case
AUTOMATIC SCALE (Y,N) : |

After running all machines, statistics are displayed and the perturbed tunes are plotted:

AUTOMATIC SCALES (Y,N) :

When the default value Y is entered, the scales of the plots are automatically calculated, according to the computed minimum and maximum values. On the other hand (N), the proposed scales can be modified:

NUX MIN , MAX : ν_x range
NUZ MIN , MAX : ν_z range

DEFD : GENERATION OF RANDOM DIPOLE ERRORS

This option deals with dipole errors. Various types of errors can be generated and analysed in the same way as in the **DEFQ** option. The input data specifying the kinds of errors to be run are the following:

<i>SIGMA DB/B</i>	: field error standard deviation $\Delta B/B$
<i>SIGMA DL/L</i>	: length error standard deviation $\Delta B/B$
<i>SIGMA S</i>	: longitudinal positioning error standard deviation [m]
<i>SIGMA X</i>	: horizontal positioning error standard deviation [m]
<i>SIGMA Z</i>	: vertical positioning error standard deviation [m]
<i>SIGMA ROT S</i>	: dipole rotation with respect to the s axis [rad]
<i>SIGMA ROT X</i>	: dipole rotation with respect to the x axis [rad]
<i>SIGMA ROT Z</i>	: dipole rotation with respect to the z axis [rad]
<i>D index</i>	: field index error
<i>D epsil</i>	: rotation of entrance and exit faces [rad]

Additional input data as well as possible plots are defined in the same way as in the **DEFQ** option.

DEFM : GENERATION OF RANDOM MULTIPLE ERRORS

This option deals with multipole errors. Position errors will affect all the elements with type **SX**, **LD** or **LT**, with uniform r.m.s. values for all the elements. Integrated strength errors will affect only the element types **LD** and **LT**, with a r.m.s. value defined independently for each element. The input data specifying the kinds of errors to be run are the following:

<i>SIGMA X</i>	: transverse displacement standard deviation [m]
<i>SIGMA Z</i>	: vertical displacement standard deviation [m]
<i>RANDOM STRENGTH ERROR (N, Y)</i>	: integrated strength error. The average value is the parameter #1 in the definition of the element, the r.m.s. value is the parameter #3
<i>SIGMA DS</i>	: azimuthal displacement standard deviation [m]
<i>SIGMA ROT S</i>	: rotation around the s axis [rad]

Additional input data as well as possible plots are defined in the same way as in the **DEFQ** option.

AL : EFFECT OF VIBRATIONS

Vibrations of the ground induce a displacement of the magnetic axis of the quadrupoles, and a closed orbit displacement. The analytical treatment of the effect of vibrations, as used in **BETA**, is developed in [7]. This option generates different sets of quadrupole displacements, computes the 1st order closed orbit "emittance" defined as :

$$\varepsilon_{c.o.} = \gamma x_{c.o.}^2 + 2\alpha x_{c.o.}x'_{c.o.} + \beta x'_{c.o.}^2$$

Two different modes allow to generate either random displacements of the quadrupoles (non-correlated local sources of noise) or displacements corresponding to a plane wave propagating across the machine (source far away from the machine without damping). This option fails if it is called after a tracking option has been performed (**XXP**, **ZZP**, **XZ**, **DEFQ**, **DEFD**). **MO** is then required to restore the adequate structure.

OPTION (HA,FR,GR,VI,%%) :

HA Generates several sets of random displacements with a given r.m.s. value, computes the 1st order closed orbit for each set, draws the variation of the average closed orbit emittance with the number of different sets.

NUMBER OF SAMPLES : number of different sets of random displacements

R.M.S. DISPLACEMENT : horizontal and vertical displacement of all quadrupoles

EMITTANCE MIN,MAX : scales for the emittance graph.

Finally, this mode plots the position of the different closed orbits, and gives the r.m.s. values of $x_{c.o.}$ and $x'_{c.o.}$. The results are expressed, in horizontal (left column) and vertical (right column) planes, as:

Eps/pi average closed orbit emittance:

$$\varepsilon_x / \pi = \frac{1}{n} \sqrt{\sum x^2 \sum x'^2 - (\sum x x')^2}$$

Beta, Alpha:

$$\beta_x = \frac{\sum x^2}{\sqrt{\sum x^2 \sum x'^2 - (\sum x x')^2}} \quad \alpha_x = \frac{-\sum x x'}{\sqrt{\sum x^2 \sum x'^2 - (\sum x x')^2}}$$

SX, SXp: r.m.s. values of $x_{c.o.}$ and $x'_{c.o.}$

$$\sigma_x = \sqrt{\sum x^2 / n}, \quad \sigma_{x'} = \sqrt{\sum x'^2 / n}$$

Displ.: average and r.m.s. values of the generated random displacements (check)

where n is the number of samples, x and x' the horizontal or vertical coordinates of the closed orbit.

FR Here the displacement of the quadrupoles corresponds to a plane shear wave propagating across the machine with a constant given velocity. The program computes the 1st order closed orbit for several frequencies of the wave, and at each frequency for several phases of the wave with respect to the machine. As, at 1st order, there is no correlation between horizontal and vertical planes, horizontal and vertical waves with the same amplitude are simultaneously tested. The emittance of the closed orbit is then computed at all points of the lattice where a kick (element KI) has been inserted and the maximum value is kept. This implies that a special structure, with kick elements (zero-deviation) at every point where the emittance growth has to be checked, has been prepared. The closed orbit emittances as a function of the frequency of the wave are plotted and the results are printed on the current log file.

FREQUENCY (Hz) MIN,MAX,STEP : data for the frequency scan
PHASE (rad) MIN,MAX,STEP : data for the phase scan
PEAK AMPLITUDE OF THE WAVE (m) : data for the wave
PROPAGATION VELOCITY (m/s) :
EMITTANCE MIN,MAX : scales for the emittance graph

Execution may be stopped by a CTRL-C command; it may be restarted or definitively stopped according to the answer to the question:

CONTINUE ? (Y,N) :

GR After FR, this allows to redraw the closed orbit emittances with different scales

OPTION (H,V) : horizontal or vertical closed orbit
OPTION (EMI,TOL) : the resulting closed orbit emittance or the maximum tolerable amplitude of the vibration can be plotted

For EMI, the axis are then entered :

AXIS (FMIN,FMAX,YMIN,YMAX) : frequency and c.o. emittance range

For TOL, the following data are needed :

LOG PLOT (Y,N) : a logarithmic or standard vertical scale can be used
MAX. TOLERABLE EMITTANCE : maximum beam emittance ϵ_{\max} according to a given emittance growth
AXIS (FMIN,FMAX,YMIN,YMAX) : frequency range and tolerable vibration amplitude (normalized to the peak amplitude of the wave used in the calculation)

The vibration amplitude associated with the defined emittance growth is computed as $\sqrt{\epsilon_{\max} / \epsilon_{c.o.}}$ and plotted

VI use of the graphic editor

%% Exit

AJ : LINEAR LATTICE MATCHING

In this option, fits of first-order lattice characteristics are performed. Provision is made to vary some of the physical parameters of the elements in the lattice and to impose various constraints on the design at specified locations. One may fit the following quantities:

- elements of the first-order transfer matrix T for one superperiod;
- characteristic functions $\beta_x, \alpha_x, \gamma_x, \beta_z, \alpha_z, \gamma_z$;
- horizontal and vertical phase advances. Only the fractional part of the phase advances measured in units of 2π has to be specified. Matching of absolute phase advances (from the starting point of the lattice to a given location) or relative phase advances (from <location 1> to <location 2>) is performed. The phase advances are related to one superperiod, except when the matching is performed from the starting point of the lattice to the end. In that case, the fractional part of the tunes for the whole machine is fitted;
- beam emittance (for use in the case of an electron or positron ring only).

The emittance is expressed as (x stands for x as well as for z):

$$E/\Gamma^2 = C_Q \langle H_x/\rho^3 \rangle / \langle 1/\rho^2 \rangle / J_x$$

where H is the chromatic invariant $H_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$, J_x is the horizontal damping partition number and $C_Q = 3.84 \cdot 10^{-13}$ m.

The matching procedure will try to fulfil the constraints by minimizing a function which is the sum of the square of all errors (defined as the difference between the desired value and the current value), each divided by a specified weight. Thus, the smaller the weight, the more important the constraint becomes. The weights are set to specified values and remain valid until changed again.

For each variable parameter, the range of variation as well as the initial step size for varying the parameter are imposed in the program, according to the initial value of the variable. So it is with the maximum number of iterations.

The minimization method is the following: at each iteration the best direction of variation and the step size (which is divided if the initial one is not efficient) is determined for each variable. A global variation is then applied, according to the individual estimates, until the minimization fails. Then a new search of the best direction of variation is tried; if the process is not successful, all the step sizes are divided and a new iteration starts. The current and the best value of the function to minimize and the variable number (0 for the global variation) are displayed at each step of variation.

In principle, any parameter of an element in the lattice may be varied. Once a variable element has been specified through its code number, all the elements having the same type are varied along the structure. It is possible to apply coupled variations to 2 different elements, so that the sum of the variable parameters is kept constant. In that case, the meaningful parameters allowed to vary are the length of drift spaces or quadrupoles, the angle of dipoles, the deflection angle of kicks.

Prior to running the option, the input data to be entered are as follows:

PRINT OUT OF THE STRUCTURE (N / Y) : if Y is entered, the list of individual elements and the structure are displayed;

NUMBER OF VARIABLES: up to 20 variables are possible; for each variable, one has to specify:

ELEM *PARAM* *COUPL*

ELEM is the number of the element to be varied;

PARAM is the number of the parameter; it has to be set to a negative value if the parameter is coupled to another one;

COUPL is omitted if a positive value has been given to *PARAM*; otherwise, the number of the coupled element is entered.

NUMBER OF CONSTRAINTS : up to 20 constraints can be defined. For each constraint, one needs

TYPE *I* *J* *NUM VAL* *DVAL*

TYPE refers to the type of constraint:

0 for characteristics functions

1 for transfer matrix

2 for phase advances (tunes)

5 for beam emittance

6 dispersion

3 and 4 are meaningless for 1st order fits

I and *J* refer to different parameters according to the value of *TYPE* :

if *TYPE* =0 the {*i,j*} beam matrix element (example: 1,1 for β_x ; 3,3 for β_z)

if *TYPE* =1 the [*i,j*] transfer matrix element ($1 \leq I, J \leq 7$)

if *TYPE* =2 *I* = 1(horizontal), 3 (vertical) for absolute and relative fits

for absolute phase advances (tunes) : *I*=1 or 3, *J*=0 , *NUM*=number of elements in structure

for relative phase advance fits from <location 1> to <location 2>, 2 constraints are needed, the first one being related to the absolute phase advance from the starting point in the lattice to <location 1>:

I *J* =0 *NUM*=<location1>

the second constraint related to the starting and end point of relative phase advance:

I *J* =<location 1> *NUM*=<location2>

if *TYPE*=5 *I* and *J* are set to 0

if *TYPE*=6 *I* =1 (η_x), 2 (η'_x), 3 (η_z), 4 (η'_z), *J* is set to 0.

NUM is the position of an element in the lattice at the end of which the constraint applies. If *TYPE*=2 and relative phase advances are specified *NUM* is chosen as is it is indicated just above.

VAL is the desired value

DVAL is the weight value. In the case of a relative phase advance fit, the weight value related to the first constraint has to be set to a large value, so that the constraint is not effective.

Example of constraints for a phase advance fit: horizontal phase advance at element 30 in the structure equal to 2.34; vertical phase advance from element 10 to element 15 equal to 0.15

```
2 1 0 30 0.34 1. absolute horizontal phase advance (tune) fit
2 3 0 10 ***** 100000. (VAL may have any value) relative vertical phase advance fit
2 3 10 15 0.15 1. relative vertical phase advance fit
```

Once all the constraints have been specified, the program performs a few checks of the data set and rearrange the input data in a well-ordered form. Since all inconsistent data cannot be checked, some meaningless fits might be run without setting alarms from the program.

Once a fit is terminated, the program displays the new lattice characteristics (transfer matrix, Twiss parameters, betatron tunes, etc). If the fit is interrupted (by a CTRL-C command), the results of the fit (actual values of the variables and the constraints) are displayed and the following questions are asked:

MORE PRECISION (Y / N) : Y causes the fit to proceed. If N is entered:

PRINT OUT OF THE RESULTS (N / Y) : Y copies the results of the fit on the BETALOG file

AJ2 : SECOND-ORDER MATCHING

This command serves the following purposes:

- to adjust second-order characteristics at desired values, namely the horizontal and the vertical chromaticities (ξ_x and ξ_z) and the dependence of α_x , β_x , α_z , β_z on momentum;
- to adjust driving terms of third-order resonances induced by sextupoles and minimize tune shifts with amplitude; details on the mathematical derivation of these terms are given in Appendix A;
- to optimize the dynamic acceptance, either in the horizontal direction or the vertical one.

In the first mode, second-order transfer is required. The two other modes are run with first-order transfer.

The input data have the form:

PRINT OUT OF THE STRUCTURE (N / Y) :if Y is entered, the list of individual elements and the structure are displayed;

NUMBER OF VARIABLES : up to 20 variables are possible; for each variable, one has to specify:

ELEM PARAM COUPL the definitions are the same as in the **AJ** option.

NUMBER OF CONSTRAINTS : up to 20 constraints can be defined. For each constraint, one needs:

TYPE *I* *J* *NUM VAL* *DVAL*

TYPE is equal to: 3 for 2nd order terms, driving terms, tune shifts with amplitude
 4 for dynamic aperture

If *TYPE* = 3, the following values of *I* and *J* are required:

I 1 for ξ_x , 2 for ξ_z , 3 for $\delta\beta_x/(\delta p/p)$, 4 for $\delta\alpha_x/(\delta p/p)$, 5 $\delta\beta_z/(\delta p/p)$, 6 for $\delta\alpha_z/(\delta p/p)$,
 (2nd order transfer)
 7 for $\delta v_x/E_x$, 8 for $\delta v_x/E_z$, 9 for $\delta v_z/E_x$, 10 for $\delta v_z/E_z$, (1st order transfer)
 14 for D, 15 for F, 16 for G, 17 for DG, 18 for L, 19 for H (1st order transfer)

J 0 for 2nd-order fits
 NHARM for driving terms and tune shifts fits

If *TYPE* = 4 *I* = 1 *J* = 1 for horizontal acceptance fit
 I = 3 *J* = 3 for vertical acceptance fit

NUM = 0 for all constraints.

VAL and *DVAL* have the same meaning as in **AJ** (for the acceptance fit, *VAL* is meaningless)

For the dynamic aperture fit, additional parameters are needed, i.e.

X INITIAL VALUE :
X STEP :
X REF :
Z INITIAL VALUE :
Z STEP :
Z REF :
NUMBER OF TURNS :
MAX. BLOW-UP :

They are defined in the same way as in **XZ**.

Execution of the option and outputs are performed as in **AJ**

SX : PLOTS OF DRIVING TERMS OF THIRD-ORDER RESONANCES

This option performs plots of driving terms of third-order resonances excited by sextupoles, namely:

$$\nu_x = p \quad (\beta_x^{3/2} \text{ term})$$

$$3 \nu_x = p$$

$$\nu_x = p \quad (\beta_x^{1/2} \beta_z \text{ term})$$

$$\nu_x - 2 \nu_z = p$$

$$\nu_x + 2 \nu_z = p$$

where p is an integer.

- for a given resonance excited by sextupoles (i.e. a fixed value of p is considered), the contributions of all sextupoles in the lattice to the corresponding driving term are plotted in a circular diagram, as well as the resultant vector and the amplitude of the driving term;
- plot of the amplitudes of driving terms for one of the listed resonances (all values of p are considered). According to the working point ν_x, ν_z , the nearest resonance corresponds to a given value p_0 . A given number of harmonics about the fundamental p_0 is drawn.

PLOT (SE, HA, %%) :

SE plots the contribution of all sextupoles to a resonance driving term

HA plots the amplitudes of driving terms

For the SE plot the following data are needed:

MAX. VERT. SCALE : scale for the circular diagram

SCALING FACTOR : this parameter is needed to adapt the magnitude of the driving term to the scale of the graph

RESONANCE $m\nu_x + n\nu_z = p$ *m, n, p* : m, n, p values

AXIS (Y, N) : if the answer is N, the axis are not drawn

According to the input values m, n, p, the contribution of sextupoles to a D_p or G_p ($m=1, n=0$), F_p ($m=3, n=0$), H_p ($m=1, n=-2$), L_p ($m=1, n=2$) term is computed.

For the HA plot, input data are the following:

PLOT (D, F, G, H, L, DG) : choice of the resonance driving terms to be plotted

NB OF HARMONICS : p_{\max} terms are calculated about the fundamental harmonic

HOR SCALE MIN, MAX : initial and final harmonic to be plotted

VERT SCALE : maximum amplitude of the driving terms

SXLD : SEXTUPOLE SUBSTITUTION
LDSX

The option **SXLD** changes the structure of the lattice by replacing all the sextupole elements (type **SX**) by thin lens multipoles (type **LD** with 6 poles). The drift spaces on both sides of the sextupoles are not modified, so this assumes that the sextupoles were previously defined with an infinitesimal length. Then, in all the tracking options (**XXP,ZZP,LDP,XZ**), the results of the two different descriptions of the element are in perfect agreement. As the storage of sextupole parameters requires much memory space, their number is limited (20 different single elements, 500 sextupoles per superperiod). The description as thin lens multipoles allows to overcome this limit. However, the description as sextupoles is necessary for all options involving the second order transfer (2nd order transfer matrices, 2nd order fits, driving terms of 3rd order resonances etc.).

On the contrary, **LDSX** will replace all the 6-pole multipoles by sextupoles with a length of 1.E-6 m and an equivalent strength.

AVAR : SENSE OF DESCRIPTION OF THE STRUCTURE

This command causes the list of single elements in the structure to be reverted. At first call, the starting point is the structure input as defined in the options **DE**, **MO** or **FI**. Afterwards, change of direction is performed on the current structure.

After running this option, all the options in **BETA** take into account the new order of description of the lattice, except **AJ** which sets the list of elements back to its initial order.

7. ACKNOWLEDGEMENTS

The present version of **BETA**, described in this report, includes some routines that are specific of electron machines. But the major part of the code was developed in Saclay (France) by the LNS Theory Group, as an improved version of a previous computer code [1]. The analytical approach and programming were completed with the contributions of T. Aniel, J.M. Filhol, E. Grorud, G. Leleux, J. Payet, D. Poirier, A. Tkatchenko.

The further development has been continued in two branches, one development branch at Saclay (J. Payet), the other at the ESRF. The here presented code is the version of the ESRF. The differences of the two codes are substantial. To obtain the version of Saclay the user should address to J. Payet (jpayet@cea.fr).

The authors wish to thank all their colleagues for giving them so much valuable help and beneficial comments.

8. REFERENCES

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APPENDIX A

In this appendix, we summarize very briefly the calculations by G. Leleux [8]. A detailed review of the analytical derivations can be found in [9].

When analysing the motion of particles in the presence of sextupoles with the Hamiltonian formalism, the Hamiltonian is expressed in the following way:

$$H = \frac{1}{2} p_x^2 + \frac{1}{2} p_z^2 + \frac{1}{2} k_x(\theta) x^2 + \frac{1}{2} k_z(\theta) z^2 + V(\theta) (x^3 - 3xz^2)$$

where

$$V(\theta) = \frac{R^2}{3! (B\rho)} \frac{\partial^2 B_z(\theta)}{\partial x^2}$$

$k_{x,z}(\theta)$ describes the focusing along the machine

$B\rho$ is the magnetic rigidity and R is the machine radius

The solutions of the unperturbed motion ($V(\theta) = 0$) are given by:

$$x = \sqrt{\frac{2 \beta_x \rho_x}{R}} \sin(\Phi_x + \phi_x)$$

$$p_x = \sqrt{\frac{2 \rho_x R}{\beta_x}} [\cos(\Phi_x + \phi_x) - \alpha_x \sin(\Phi_x + \phi_x)]$$

$$z = \sqrt{\frac{2 \beta_z \rho_z}{R}} \sin(\Phi_z + \phi_z)$$

$$p_z = \sqrt{\frac{2 \rho_z R}{\beta_z}} [\cos(\Phi_z + \phi_z) - \alpha_z \sin(\Phi_z + \phi_z)]$$

where

$$\Phi_{x,z} = R \int_0^\theta \frac{d\theta}{\beta_{x,z}(\theta)}$$

$$\rho_{x,z} = \frac{R}{2} \frac{\epsilon_{x,z}}{\pi}$$

and $\epsilon_{x,z}$ are the transverse emittances

By applying the following canonical transformation $x, p_x, z, p_z \rightarrow \phi_x, \rho_x, \phi_z, \rho_z$, the Hamiltonian reduces to:

$$\Delta H = V(\theta) \left\{ \left(\frac{2\beta_x \rho_x}{R} \right)^{\frac{3}{2}} \sin^3(\Phi_x + \phi_x) - 3 \left(\frac{2\beta_x \rho_x}{R} \right)^{\frac{1}{2}} \left(\frac{2\beta_z \rho_z}{R} \right) \sin(\Phi_x + \phi_x) \sin^2(\Phi_z + \phi_z) \right\}$$

A further canonical transformation $\phi_{x,z}, \rho_{x,z} \rightarrow \bar{\phi}_{x,z}, \rho_{x,z}$ is used to draw the periodical terms. The Hamiltonian can then be rewritten as:

$$\begin{aligned} \Delta H = & v_x \rho_z + v_z \rho_z + \frac{3}{4} V(\theta) \left(\frac{2\beta_x}{R} \right)^{\frac{3}{2}} \rho_x^{\frac{3}{2}} \sin(\psi_x + \bar{\phi}_x) \\ & - \frac{1}{4} V(\theta) \left(\frac{2\beta_x}{R} \right)^{\frac{3}{2}} \rho_x^{\frac{3}{2}} \sin 3(\psi_x + \bar{\phi}_x) \\ & - \frac{3}{2} V(\theta) \left(\frac{2\beta_x}{R} \right)^{\frac{1}{2}} \left(\frac{2\beta_z}{R} \right) \rho_x^{\frac{1}{2}} \rho_z \sin(\psi_x + \bar{\phi}_x) \\ & + \frac{3}{4} V(\theta) \left(\frac{2\beta_x}{R} \right)^{\frac{1}{2}} \left(\frac{2\beta_z}{R} \right) \rho_x^{\frac{1}{2}} \rho_z \sin(\psi_x - 2\psi_z + \bar{\phi}_x - 2\bar{\phi}_z) \\ & + \frac{3}{4} V(\theta) \left(\frac{2\beta_x}{R} \right)^{\frac{1}{2}} \left(\frac{2\beta_z}{R} \right) \rho_x^{\frac{1}{2}} \rho_z \sin(\psi_x + 2\psi_z + \bar{\phi}_x + 2\bar{\phi}_z) \end{aligned}$$

where $\Psi_{x,z} = \Phi_{x,z} - v_{x,z}\theta$

$$\bar{\phi}_{x,z} = \Phi_{x,z} + v_{x,z}\theta$$

One can analyse in Fourier's series the terms which are periodic over 2π . The Hamiltonian then reduces to:

$$\begin{aligned}\Delta H = & v_x \rho_x + v_z \rho_z + \sum_m |D_m| \rho_x^{3/2} \sin(\bar{\phi}_x - m\theta + d_m) \\ & + \sum_n |F_n| \rho_x^{3/2} \sin(3\bar{\phi}_x - n\theta + f_n) \\ & + \sum_p |G_p| \rho_x^{1/2} \rho_z \sin(\bar{\phi}_x - p\theta + g_p) \\ & + \sum_q |H_q| \rho_x^{1/2} \rho_z \sin(\bar{\phi}_x - 2\bar{\phi}_z - q\theta + h_q) \\ & + \sum_r |L_r| \rho_x^{1/2} \rho_z \sin(\bar{\phi}_x + 2\bar{\phi}_z - r\theta + l_r)\end{aligned}$$

Starting from this Hamiltonian one can calculate $\frac{d\bar{\phi}_x}{d\theta}$ and $\frac{d\bar{\phi}_z}{d\theta}$:

$$\frac{d\bar{\phi}_x}{d\theta} = \frac{\partial \Delta H}{\partial \rho_x} = v_x + \sum_m \frac{3}{2} |D_m| \rho_x^{1/2} \sin(\bar{\phi}_x - m\theta + d_m) + \dots$$

$$\frac{d\bar{\phi}_z}{d\theta} = \frac{\partial \Delta H}{\partial \rho_z} = v_z + \sum_p |G_p| \rho_x^{1/2} \sin(\bar{\phi}_x - p\theta + g_p) + \dots$$

Since, in these analytical expressions, some terms cannot be solved as a function of θ , an approximate treatment is proposed to derive the tune shifts with amplitude Δv_x and Δv_z :

- assuming that the phases $\phi_{x,z}$ and the amplitudes $\rho_{x,z}$ are constant, a first approximation integration of the equations can be performed; we obtain:

$$\phi_{x,z} = (\phi_{x,z})_0 + \dots$$

$$\rho_{x,z} = (\rho_{x,z})_0 + \dots$$

- we put these first-order values back into $\frac{d\bar{\phi}_x}{d\theta}$ and $\frac{d\bar{\phi}_z}{d\theta}$
- by averaging over all the initial phases $(\phi_x)_0$ and $(\phi_z)_0$, the tune shifts with amplitude then are given by:

$$\Delta v_x = -\frac{3}{4} \left\{ \sum_m \frac{|D_m|^2}{v_x - m} + 3 \sum_n \frac{|F_n|^2}{3v_x - n} \right\} \frac{\epsilon_x}{\pi}$$

$$+ \frac{1}{2} \left\{ \sum_q \frac{|H_q|^2}{v_x - 2v_z - q} - \sum_r \frac{|L_r|^2}{v_x + 2v_z - r} - \sum_n \frac{|D_m||G_m|}{v_x - n} \cos(d_m - g_m) \right\} \frac{\epsilon_z}{\pi}$$

$$\Delta v_z = \frac{1}{2} \left\{ \sum_q \frac{|H_q|^2}{v_x - 2v_z - q} - \sum_r \frac{|L_r|^2}{v_x + 2v_z - r} - \sum_n \frac{|D_m||G_m|}{v_x - n} \cos(d_m - g_m) \right\} \frac{\epsilon_x}{\pi}$$

$$- \frac{1}{4} \left\{ \sum_p \frac{|G_p|^2}{v_x - p} + \sum_q \frac{|H_q|^2}{v_x - 2v_z - q} + \sum_r \frac{|L_r|^2}{v_x + 2v_z - r} \right\} \frac{\epsilon_z}{\pi}$$

The optimization of sextupoles in order to minimize the third-order resonances driving terms and the tune shifts with amplitude is based on this formalism. For a given working point v_x, v_z , the process is the following:

- assuming that the sextupoles are thin lenses at locations $\theta = \theta_i$ in the machine, the coefficients D_m, F_n, G_p, H_q, L_r are given by:

$$D_m = \frac{1}{8\pi} \sum_i (H)_i (2\beta_x)_i^{\frac{3}{2}} e^{j [\Phi_{x_i} + (m - v_x) \theta_i]}$$

$$F_n = -\frac{1}{24\pi} \sum_i (H)_i (2\beta_x)_i^{\frac{3}{2}} e^{j \{ 3 \Phi_{x_i} + (n - 3 v_x) \theta_i \}}$$

$$G_p = -\frac{1}{4\pi} \sum_i (H)_i (2\beta_x)_i^{\frac{1}{2}} (2\beta_z)_i e^{j [\Phi_{x_i} + (p - v_x) \theta_i]}$$

$$H_q = \frac{1}{8\pi} \sum_i (H)_i (2\beta_x)_i^{\frac{1}{2}} (2\beta_z)_i e^{j [\Phi_{x_i} - 2\Phi_{z_i} + (q - v_x + 2v_z) \theta_i]}$$

$$L_r = \frac{1}{8\pi} \sum_i (HI)_i \frac{1}{(2\beta_x)_i^2 (2\beta_z)_i} e^{j [\Phi_{x_i} + 2\Phi_{z_i} + (r - \nu_x - 2\nu_z) \theta_i]}$$

where $(HI)_i$ is the strength of sextupole i

$$\Phi_{x,z} = \int \frac{ds}{\beta_{x,z}}$$

Using the cosine and sine values of the coefficients which are summed up, it is possible to plot in a circular diagram the contributions of all sextupoles to a given driving term.

The amplitudes of the driving terms of the third-order resonances can be compared through the values of the different harmonics, namely:

$$\frac{|D_m|^2}{\nu_x - m}, \quad \frac{|F_n|^2}{3\nu_x - n}, \quad \frac{|G_p|^2}{\nu_x - p}, \quad \frac{|H_q|^2}{\nu_x - 2\nu_z - q}, \quad \frac{|L_r|^2}{\nu_x + 2\nu_z - r}$$

- the different sums over a given number of harmonics are calculated about the fundamental harmonic. For each term, the resonance nearest to the machine tunes determines the fundamental harmonic m_0, n_0, p_0, q_0, r_0 .

In order to determine a proper sextupole configuration, the program can be asked to minimize for each kind of resonance the fundamental harmonic and the sum over a given number of harmonics and simultaneously to reduce the values of tune shifts with amplitude.

APPENDIX B

An example of run for the ESRF storage ring [10] is given in this section. The most usual input and output formats of **BETA** are illustrated by attached figures (for all figures the structure `nom7nm_pc.str` was used) .

- Figure 1 : Input file `nom7nm.str` displayed when running the option **MO**.
- Figure 2 : Output resulting of the transfer in the structure after running **DE, MO, FI, X**.
- Figure 3 : Optical functions (option **GR**).
- Figure 4 : Calculation of chromatic closed orbit and plot of ν_x as a function of momentum (option **NUDP**).
- Figure 5 : Input data for particle tracking in the horizontal phase plane (option **XXP**).
- Figure 6 : Particle tracking in the horizontal phase plane and Fourier analysis (option **XXP**).
- Figure 7 : Dynamic aperture (option **XZ**).
- Figure 8 : Generation of random quadrupole positioning errors, calculation of the resulting closed orbit, calculation (option **DEFQ**).
- Figure 9 : Plot of the perturbed tunes (option **DEFQ**).
- Figure 10 : Input data for a phase advance fit and display of the results (option **AJ**).
- Figure 11 : Input data for a second order fit of chromaticity, driving terms and tune shifts and display of the results (option **AJ2**).
- Figure 12 : Plot of the contribution of sextupoles to a 3rd order resonance driving term (option **SX**).
- Figure 13 : Plot of the amplitudes of driving terms (option **SX**).
- Figure 14 : Tune diagram for one superperiod with systematic regular third-order and 5th resonance (option **DIAG**).

OPTION(%%, \$, Z, VI, AX, X, DE, MO, FI, DIAG, SY, GR, TR, CO,
 NUDD, XXP, ZZP, LDP, XZ, EXZ, DEFQ, DEFD, DEFM, AL
 AJ, AJ2, SX, SXLD, LDSX, AVAR, NLOG, MO2) : mo

Structure as RING(BETA) or TRANSFER LINE(BEAM) (RI/TL) [RI] :

```

63
  1 SD .61051998E+01
QD1 QP .43000001E+00 -.28946999E+00
  3 SD .38299999E+00
S4 SX .10000000E-05 .15900000E+07
QF2 QP .93000001E+00 .51270998E+00
S6 SX .10000000E-05 -.15700000E+07
S0 SD .34999999E+00
QD3 QP .52999997E+00 -.50567001E+00
  9 SD .10722500E+01
 10 CO .49087401E-01 .00000000E+00 .00000000E+00 .00000000E+00
 11 DI .92324801E-01 .23366301E+02 .00000000E+00 .00000000E+00
 12 CO .43237399E-01 .00000000E+00 .00000000E+00 .00000000E+00
 13 CO -.43237399E-01 .00000000E+00 .00000000E+00 .00000000E+00
 14 DI .58499998E-02 .50035999E+02 .00000000E+00 .00000000E+00
 15 CO .49087401E-01 .00000000E+00 .00000000E+00 .00000000E+00
 16 CO .49087401E-01 .00000000E+00 .00000000E+00 .00000000E+00
 17 DI .58499998E-02 .50035999E+02 .00000000E+00 .00000000E+00
 18 CO -.43237399E-01 .00000000E+00 .00000000E+00 .00000000E+00
 19 CO .43237399E-01 .00000000E+00 .00000000E+00 .00000000E+00
 20 DI .92324801E-01 .23366301E+02 .00000000E+00 .00000000E+00
NEXT ? :
63
 21 CO .49087401E-01 .00000000E+00 .00000000E+00 .00000000E+00
S22 SX .10000000E-05 -.11500000E+07
 23 SD .95475000E+00
S24 SX .10000000E-05 .21200000E+07
QD4 QP .42199999E+00 -.65629798E+00
 26 SD .30000001E+00
S13 SX .10000000E-05 -.18100000E+07
 28 SD .46300000E+00
S19 SX .10000000E-05 .31300000E+07
QF5 QP .50999999E+00 .74452299E+00
 31 SD .50000000E+00
QD8 QP .42899999E+00 -.50976002E+00
QF7 QP .90600002E+00 .81853998E+00
QD6 QP .50900000E+00 -.76413798E+00
S20 SX .10000000E-05 -.17751200E+07
 36 CA .00000000E+00 .10000000E+01 .60000000E+10

```

```

PICK PU .00000000E+00 .00000000E+00
 38 KI .00000000E+00 .00000000E+00 .00000000E+00
 39 KI .00000000E+00 .00000000E+00 .30000000E+01
 40 KI .00000000E+00 .00000000E+00 .00000000E+00
NEXT ? :
 63
 41 KI .00000000E+00 .00000000E+00 .00000000E+00
C19 KI .00000000E+00 .00000000E+00 .10000000E+01
 43 KI .00000000E+00 .00000000E+00 .00000000E+00
 44 KI .00000000E+00 .00000000E+00 .30000000E+01
 45 KI .00000000E+00 .00000000E+00 .00000000E+00
 46 SD .30525999E+01
 47 SD .11700000E+00
 48 SD .28299999E+00
 49 SD .43000001E+00
SS1 SD .33500001E+00
SS2 SD .10200000E+00
SS3 SD .10572500E+01
SS4 SD .33899999E+00
SS5 SD .10600000E+00
SS6 SD .42500001E+00
SS7 SD .11200000E+00
SS8 SD .49500000E+00
SS9 SD .10677500E+01
SS10 SD .34549999E+00
SS11 SD .33550000E+00
NEXT ? :
 63
SS12 SD .10250000E+00
SS13 SD .11400000E+00
SS14 SD .34700000E+00
NEXT ? :
 139
 46 PICK SS2 QD1 SS1 38 S4 SS1 QF2 SS2
PICK 48 39 S6 SS1 QD3 SS3 10 11 12
 13 14 15 23 PICK SS5 QD4 SS4 41 S13
SS6 QF5 SS7 PICK 3 C19 S19 SS8 QF5 SS6
 41 S13 SS4 QD4 SS5 PICK 23 16 17 18
 19 20 21 SS9 QD6 SS10 44 S22 48 PICK
SS13 QF7 SS14 45 S24 SS11 QD8 SS12 PICK 1
PICK SS12 QD8 SS11 45 S24 SS14 QF7 SS13 PICK
 48 44 S22 SS10 QD6 SS9 10 11 12 13
 14 15 23 PICK SS5 QD4 SS4 41 S20 SS6

```

QF5 SS7 PICK 3 C19 S19 SS8 QF5 SS6 41
 S13 SS4 QD4 SS5 PICK 23 16 17 18 19
 20 21 SS3 QD3 SS1 39 S6 48 PICK SS2
 QF2 SS1 38 S4 SS1 QD1 SS2 PICK 46
 NEXT ?:
 NUMBER OF SUPERPERIODS [16]:
 PARTICLE [E]:

Figure 1

FIRST OR SECOND ORDER(1 OU 2):
 OPTION : BETA
 STRUCTURE: D:\beta\structures\nom7nm_pc.str
 0/ 139

FIRST-ORDER MATRIX

-1.719E-01	2.912E+01	0.000E+00	0.000E+00	0.000E+00	-1.324E-03	0.000E+00
-3.333E-02	-1.719E-01	0.000E+00	0.000E+00	0.000E+00	-3.767E-05	0.000E+00
0.000E+00	0.000E+00	-2.371E-01	-1.185E+01	0.000E+00	0.000E+00	0.000E+00
0.000E+00	0.000E+00	7.965E-02	-2.371E-01	0.000E+00	0.000E+00	0.000E+00
-3.767E-05	-1.324E-03	0.000E+00	0.000E+00	1.000E+00	1.496E-02	0.000E+00
0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.000E+00	0.000E+00

MACHINE RADIUS = 134.388916
 CELL LENGTH = 52.774403

NUX= .439969 NUZ= .390390 NUS=*****

NEXT PAGE ?

2.9556E+01						
-1.0425E-15	3.3835E-02					
-5.1964E-17	-1.0373E-16	1.2198E+01				
-2.4794E-16	9.2324E-33	-1.0857E-15	8.1983E-02			
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00		
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	

CLOSED ORBIT

0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00

CHROMATIC CLOSED ORBIT

-1.1301E-03 1.0979E-16 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00

ALPHAP= 2.834E-04 ETA= 2.834E-04

W= 6.000E+09 E0= 5.110E+05

V/C= 1.000E+00 GAMMA= 1.174E+04

NEXT PAGE ?

0/ 139

	DAMPING	PARTITION	EMITTANCE		
	TIME (s)	NUMBER	INVARIANT	PROJECTION	DP/P=0
X	7.1200E-03	9.9855E-01	6.6931E-09		
Z	7.1097E-03	1.0000E+00			
S	3.5523E-03	2.0014E+00			
X	7.1200E-03		6.6753E-09	6.6753E-09	6.6753E-09
Z	7.1097E-03		1.7218E-40	1.7218E-40	1.7218E-40
	COUPLING		2.5794E-32	2.5794E-32	2.5794E-32
	ENERGY SPREAD		1.0559E-03		
	BUNCH LENGTH		0.0000E+00		
	ENERGY LOSS/TURN (MeV)		4.7543E+00		
I1			2.3932E-01		
I2			2.6036E-01		
I3			1.0972E-02		
I4			3.7737E-04		
I5			3.2863E-05		

NEXT PAGE ?

200 HARMONICS

D	.58722E+04	NUX-	0	.12096E+06	.20599E+02
G	.87861E+04	NUX-	256	-.87361E+05	-.99431E+01
DG	.52467E+05	NUX--	192	-.90888E+05	-.17323E+01
F	.30547E+04	3*NUX-	80	.20226E+05	.66213E+01
L	-.26947E+05	NUX+2*NUZ-	272	-.35666E+05	.13236E+01
H	.22718E+05	NUX-2*NUZ--	192	.39611E+05	.17436E+01

BETATRON TUNES

NUX= 36.439964 NUZ= 11.390389

TUNE SHIFTS WITH AMPLITUDE

DNUX/(EX/PI) = -.11277E+05 DNUZ/(EX/PI) = -.14010E+04
 DNUX/(EZ/PI) = -.14010E+04 DNUZ/(EZ/PI) = -.11391E+04

NEXT PAGE ?

SEXTUPOLES

NUM	HL	TETA	BETAX	PSIX	BETAZ	PSIZ
1	1.59000E+00	2.91661E-02	3.43546E+01	1.30607E-01	1.17136E+01	3.14044E-01
2	-1.57000E+00	4.14439E-02	2.10886E+01	1.81598E-01	1.78524E+01	4.49446E-01
3	-1.81000E+00	8.75340E-02	1.85120E+01	2.90114E+00	1.82696E+01	7.05060E-01
4	3.13000E+00	9.81748E-02	3.30207E+01	2.95167E+00	1.13348E+01	8.17849E-01
5	-1.81000E+00	1.08816E-01	1.85833E+01	3.00210E+00	1.84718E+01	9.30016E-01
6	-1.15000E+00	1.54906E-01	2.28304E+01	5.71444E+00	1.55390E+01	1.18253E+00
7	2.12000E+00	1.67183E-01	2.77055E+01	5.76315E+00	6.23982E+00	1.40090E+00
8	2.12000E+00	2.25516E-01	2.77055E+01	8.54679E+00	6.23982E+00	3.07210E+00
9	-1.15000E+00	2.37794E-01	2.28304E+01	8.59550E+00	1.55390E+01	3.29047E+00
10	-1.77512E+00	2.83884E-01	1.85833E+01	1.13078E+01	1.84718E+01	3.54298E+00
11	3.13000E+00	2.94524E-01	3.30207E+01	1.13583E+01	1.13348E+01	3.65515E+00
12	-1.81000E+00	3.05165E-01	1.85120E+01	1.14088E+01	1.82696E+01	3.76794E+00
13	-1.57000E+00	3.51255E-01	2.10886E+01	1.41283E+01	1.78524E+01	4.02355E+00
14	1.59000E+00	3.63533E-01	3.43546E+01	1.41793E+01	1.17136E+01	4.15895E+00

QUADRUPOLES

NUM	KL	TETA	BETAX	PSIX	BETAZ	PSIZ
1	-1.24472E-01	2.50735E-02	3.04802E+01	1.13487E-01	1.28980E+01	2.69601E-01
2	4.76820E-01	3.51190E-02	3.57949E+01	1.52265E-01	1.15050E+01	3.86782E-01
3	-2.68005E-01	4.59086E-02	1.16403E+01	2.20725E-01	2.39419E+01	4.77947E-01
4	-2.76958E-01	8.34414E-02	1.03652E+01	2.86072E+00	2.42296E+01	6.79321E-01
5	3.79707E-01	9.25940E-02	3.09601E+01	2.92883E+00	1.21106E+01	7.52025E-01
6	3.79707E-01	1.03756E-01	3.10138E+01	2.97450E+00	1.21896E+01	8.83459E-01
7	-2.76958E-01	1.12908E-01	1.04362E+01	3.04231E+00	2.45579E+01	9.55442E-01
8	-3.88946E-01	1.50441E-01	1.15294E+01	5.67636E+00	2.42401E+01	1.15239E+00
9	7.41597E-01	1.61231E-01	3.89509E+01	5.74069E+00	7.16168E+00	1.27041E+00
10	-2.18687E-01	1.71276E-01	1.75326E+01	5.78846E+00	6.55274E+00	1.48595E+00
11	-2.18687E-01	2.21423E-01	1.75326E+01	8.52149E+00	6.55274E+00	2.98705E+00
12	7.41597E-01	2.31469E-01	3.89509E+01	8.56925E+00	7.16167E+00	3.20259E+00
13	-3.88946E-01	2.42258E-01	1.15294E+01	8.63358E+00	2.42401E+01	3.32060E+00
14	-2.76958E-01	2.79791E-01	1.04362E+01	1.12676E+01	2.45579E+01	3.51755E+00
15	3.79707E-01	2.88944E-01	3.10138E+01	1.13354E+01	1.21896E+01	3.58954E+00
16	3.79707E-01	3.00105E-01	3.09601E+01	1.13811E+01	1.21106E+01	3.72097E+00
17	-2.76958E-01	3.09258E-01	1.03652E+01	1.14492E+01	2.42296E+01	3.79367E+00
18	-2.68005E-01	3.46791E-01	1.16403E+01	1.40892E+01	2.39419E+01	3.99505E+00
19	4.76820E-01	3.57580E-01	3.57949E+01	1.41577E+01	1.15050E+01	4.08621E+00
20	-1.24472E-01	3.67626E-01	3.04802E+01	1.41965E+01	1.28980E+01	4.20339E+00

Figure 2

```

OPTION(%%, $, Z, VI, AX, X, DE, MO, FI, DIAG, SY, GR, TR, CO,
      NUDP, XXP, ZZP, LDP, XZ, EXZ, DEFQ, DEFD, DEFM, AL
      AJ, AJ2, SX, SXLD, LDSX, AVAR, NLOG, MO2) :gr
OPTION(BETA, PH, ENV, TR, ENVS) :BETA
PRINT OUT OF THE VALUES (N/Y) :N
AUTOMATIC SCALES (Y/N) :Y

```

```

NUX = 36.440      R = 134.3889
NUZ = 11.390     ALPHA= 2.834E-04      OPTICAL FUNCTIONS      Ex/Gam**2= 4.854E-17

```

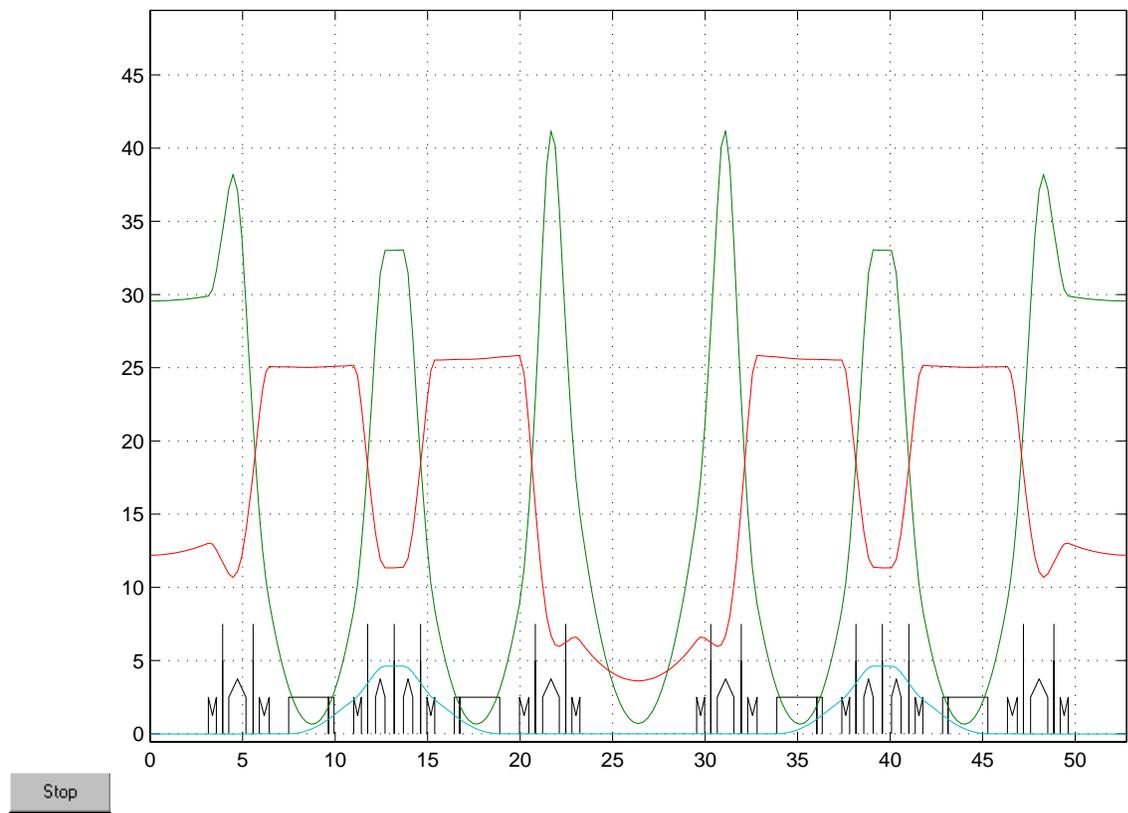


Figure 3

```

OPTION (%%, $, Z, VI, AX, X, DE, MO, FI, DIAG, SY, GR, TR, CO,
        NU DP, XXP, ZZP, LDP, XZ, EXZ, DEFQ, DEF D, DEF M, AL
        AJ, AJ2, SX, SXLD, LDSX, AVAR, NLOG, MO2) : nudp
PRECISION C.O      [   .10000E-06 ] :
NB STEPS, NB TURNS/STEP [ 200   1 ] :
DP/P MIN          [  -2.00000E-02 ] :
DP/P MAX          [   2.00000E-02 ] :
DP/P STEP         [   1.00000E-02 ] : 2.5e-3
   5   8.4764E-04  1.0449E-06  0.0000E+00  0.0000E+00 -1.3950E-02 -2.0000E-02
   5   6.4734E-04  7.6149E-07  0.0000E+00  0.0000E+00 -1.1936E-02 -1.7500E-02
   5   4.7456E-04  5.3281E-07  0.0000E+00  0.0000E+00 -1.0013E-02 -1.5000E-02
   4   3.2926E-04  3.5239E-07  0.0000E+00  0.0000E+00 -6.5394E-03 -1.2500E-02
   4   2.1116E-04  2.1510E-07  0.0000E+00  0.0000E+00 -5.1294E-03 -1.0000E-02
   4   1.1980E-04  1.1549E-07  0.0000E+00  0.0000E+00 -3.7756E-03 -7.5000E-03
   4   5.4626E-05  4.9036E-08  0.0000E+00  0.0000E+00 -2.4727E-03 -5.0000E-03
   3   1.4941E-05  1.1654E-08  0.0000E+00  0.0000E+00 -9.1179E-04 -2.5000E-03
   1   0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
   3   9.0219E-06  1.0681E-08  0.0000E+00  0.0000E+00  8.8413E-04  2.5000E-03
   4   4.1179E-05  4.1224E-08  0.0000E+00  0.0000E+00  2.3249E-03  5.0000E-03
   4   9.5649E-05  8.9027E-08  0.0000E+00  0.0000E+00  3.4418E-03  7.5000E-03
   4   1.7159E-04  1.5206E-07  0.0000E+00  0.0000E+00  4.5332E-03  1.0000E-02
   4   2.6815E-04  2.2847E-07  0.0000E+00  0.0000E+00  5.6020E-03  1.2500E-02
   4   3.8453E-04  3.1668E-07  0.0000E+00  0.0000E+00  6.6513E-03  1.5000E-02
   5   5.1988E-04  4.1562E-07  0.0000E+00  0.0000E+00  9.6049E-03  1.7500E-02
   5   6.7343E-04  5.2361E-07  0.0000E+00  0.0000E+00  1.0877E-02  2.0000E-02
PLOT (%%, NUX, BX, AX, NUZ, BZ, AZ) : nux
Y-AXIS MIN      [   3.39969E-01 ] :
Y-AXIS MAX      [   5.39969E-01 ] : 0.45

```

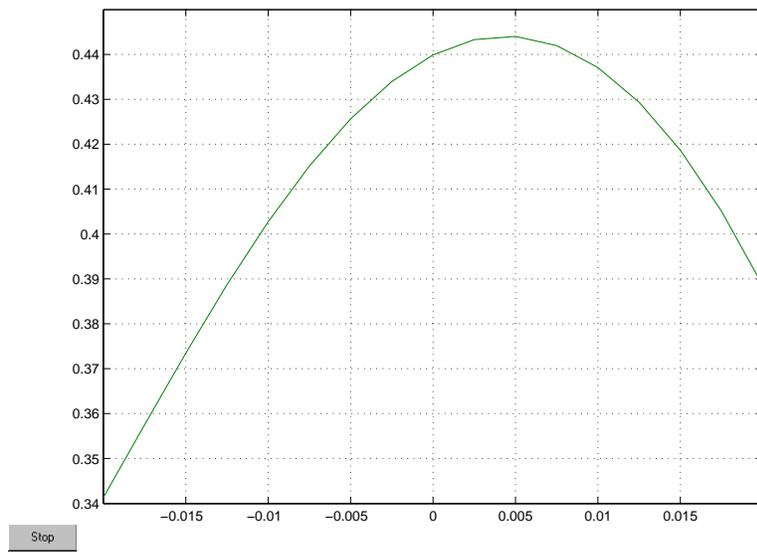


Figure 4

This illustrates the results of tracking over 1 superperiod :

```
OPTION(%%,$,Z,VI,AX,X,DE,MO,FI,DIAG,SY,GR,TR,CO,
      NU,DP,XXP,ZZP,LDP,XZ,EXZ,DEFQ,DEFD,DEFM,AL
      AJ,AJ2,SX,SXLD,LDSX,AVAR,NLOG,MO2) : xxp
****      CHROMATIC CLOSED ORBIT      ****
DP/P          [ 0.00000E+00] :
PRECISION C.O [ .10000E-06] :
NB STEPS, NB TURNS/STEP [ 200 1] :
      1      0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00
ONE PARTICLE  EX,PHX,EZ,PHZ,ES,PHS..... :EP
ONE PARTICLE  X,X',Z,Z',L,DP/P..... :YYP
NX*NZ*NS PARTICLES EX,PHX,EZ,PHZ..... :NP
C.O COORDINATES XC,X'C,ZC,Z'C,LC,DP/P.. :DE
```

```
OPTION (%%,EP,YYP,NP,DE) : yyp
PREVIOUS INITIAL BETATRON CONDITIONS
X      : .00000E+00
X'     : .00000E+00
Z      : .00000E+00
Z'     : .00000E+00
L      : .00000E+00
DP/P   : .00000E+00
PREVIOUS FINAL BETATRON CONDITIONS
X      : .00000E+00
X'     : .00000E+00
Z      : .00000E+00
Z'     : .00000E+00
L      : .00000E+00
DP/P   : .00000E+00
PREVIOUS INITIAL OR FINAL CONDITIONS (IN OR FI):IN
X      .00000E+00 : 0.01
X'     .00000E+00 :
Z      .00000E+00 :
Z'     .00000E+00 :
L      .00000E+00 :
DP/P   .00000E+00 :
.33835E-05 .00000E+00
```

```
BLOWUP TEST NO(=def), (A)BSOLUTE, (R)ELATIVE COMPARISON:
ONE PARTICLE  EX,PHX,EZ,PHZ,ES,PHS..... :EP
ONE PARTICLE  X,X',Z,Z',L,DP/P..... :YYP
```

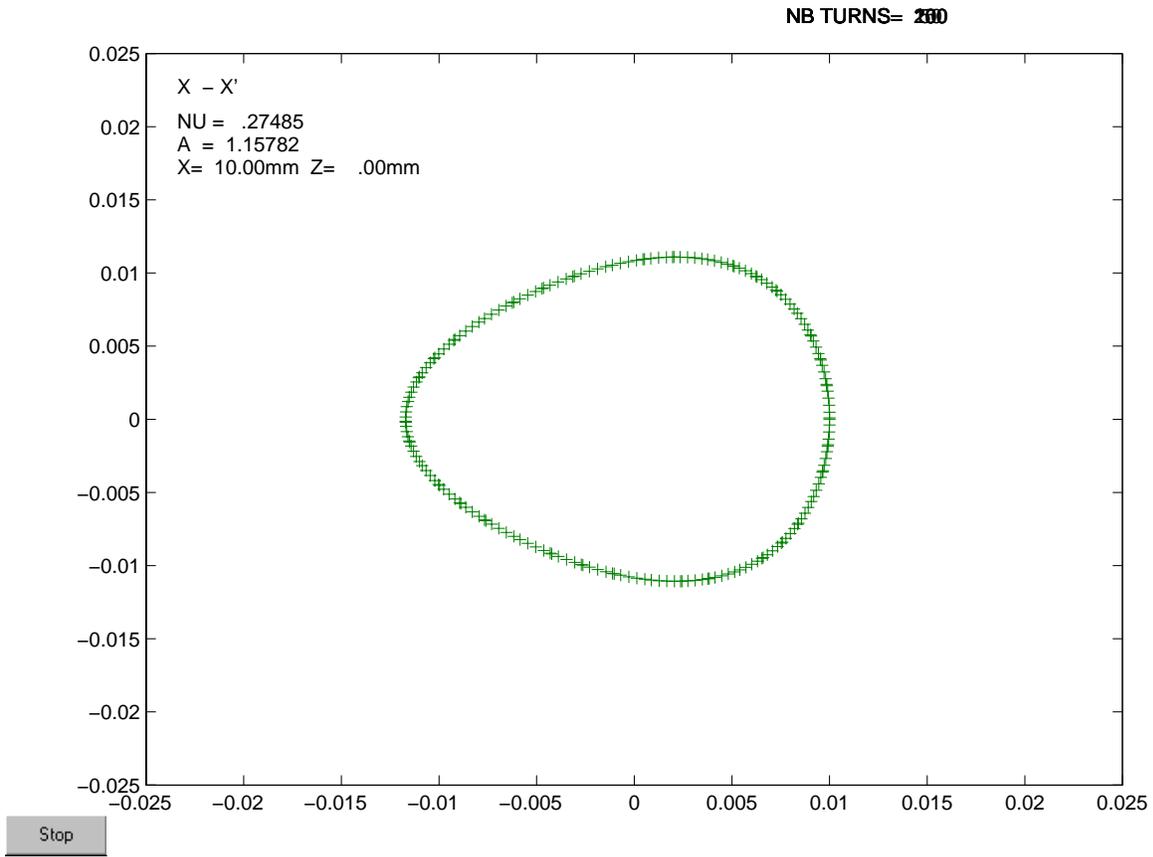
```

NX*NZ*NS PARTICLES EX,PHX,EZ,PHZ..... :NP
C.O COORDINATES XC,X'C,ZC,Z'C,LC,DP/P.. :DE

OPTION (%%,EP,YYP,NP,DE) :
NUMBER OF TURNS [ 200]:
FREQUENCY SPECTRUM RANGE
MINIMUM NU          [ 0.00000E+00]: 0.23
MAXIMUM NU          [ 5.00000E-01]: 0.29
NB OF CHANNELS      [ 11]: 100
NORMALIZED,NON-NORMALIZED VERTICAL SCALE(NO,NN) :
NORMALIZED,NON-NORMALIZED PHASE PLANE,INI. COND. (NO,NN,IC) :
EMERGENCY EXIT(NO,%%) :
      ****AXIS FOR HORIZONTAL PHASE SPACE****
MIN VALUE ON HORIZONTAL AXIS [ 0.000000E+00]: -0.025
MAX VALUE ON HORIZONTAL AXIS [ 2.500000E-02]:
MIN VALUE ON VERTICAL AXIS  [ -2.500000E-02]:
MAX VALUE ON VERTICAL AXIS  [ 2.500000E-02]:
EXIT (%%), SPECTRUM (S), NEW TRACKING (T), NEW DISPLAY (D) : s
EXIT (%%), SPECTRUM (S), NEW TRACKING (T), NEW DISPLAY (D) : %%

```

Figure 5



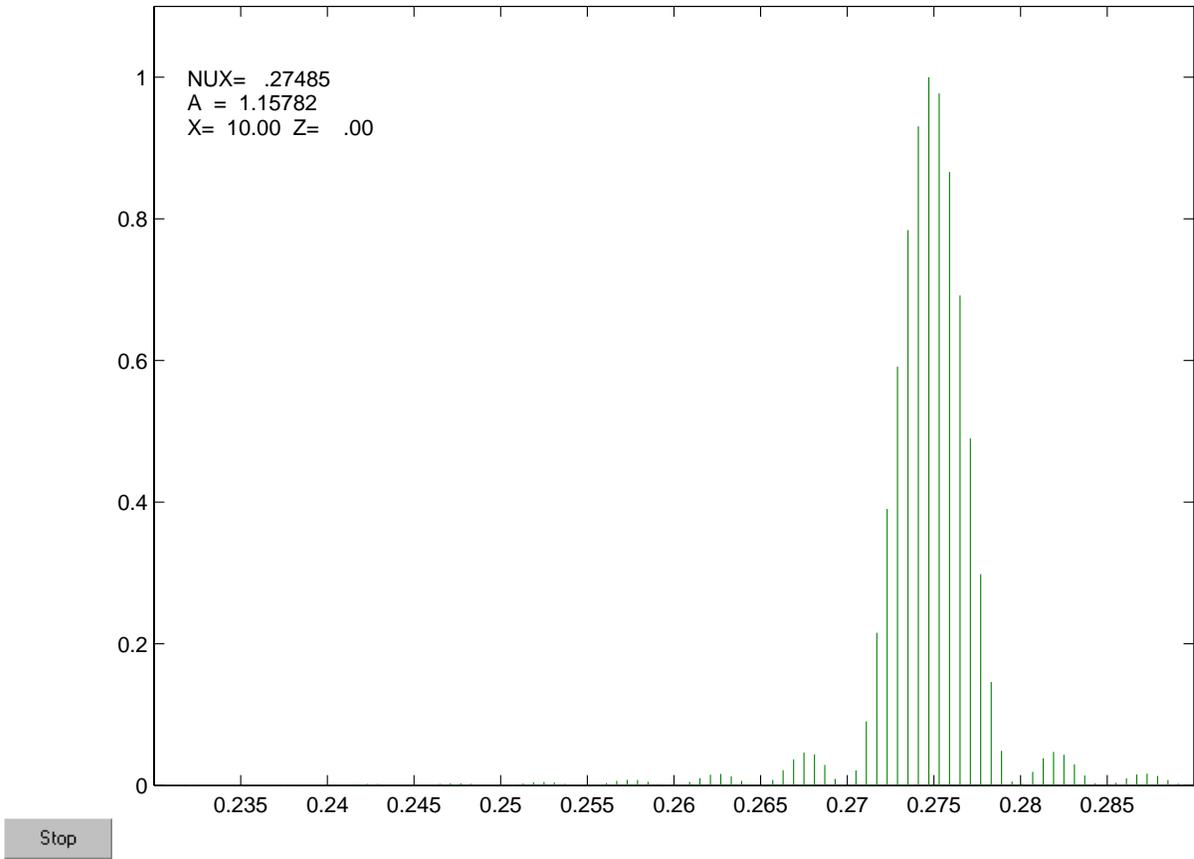


Figure 6

```

OPTION (%%, $, Z, VI, AX, X, DE, MO, FI, DIAG, SY, GR, TR, CO,
        NUDP, XXP, ZZP, LDP, XZ, EXZ, DEFQ, DEFD, DEFM, AL
        AJ, AJ2, SX, SXLD, LDSX, AVAR, NLOG, MO2) : xz
  
```

```

ANGLE (deg)  SCAN
MINIMUM ANGLE (deg) [ 0.00000E+00]:
MAXIMUM ANGLE (deg) [ 1.80000E+02]:
ANGLE (deg)  STEP  [ 4.50000E+00]:
  
```

```

AMPLITUDE    SCAN
MINIMUM AMPLITUDE [ 0.00000E+00]:
MAXIMUM AMPLITUDE [ 4.00000E-02]: 8e-2
AMPLITUDE    STEP  [ 2.00000E-03]: 4e-3
  
```

```

****    CHROMATIC CLOSED ORBIT    ****
DP/P          [ 0.00000E+00]:
PRECISION C.O [ .10000E-06]:
  
```

```
NB STEPS, NB TURNS/STEP [ 200 1]:  
1 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00  
BLOWUP TEST (N)O, (A)BSOLUTE, (R)ELATIVE COMPARISON:  
NUMBER OF TURNS [ 50]:  
LINE STYLE [ 1]:  
AUTOMATIC SCALES (Y/N) : n  
XMIN [-4.00000E-02]: -8e-2  
XMAX [ 4.00000E-02]: 8e-2  
ZMIN [ 0.00000E+00]:  
ZMAX [ 4.00000E-02]: 5e-2
```

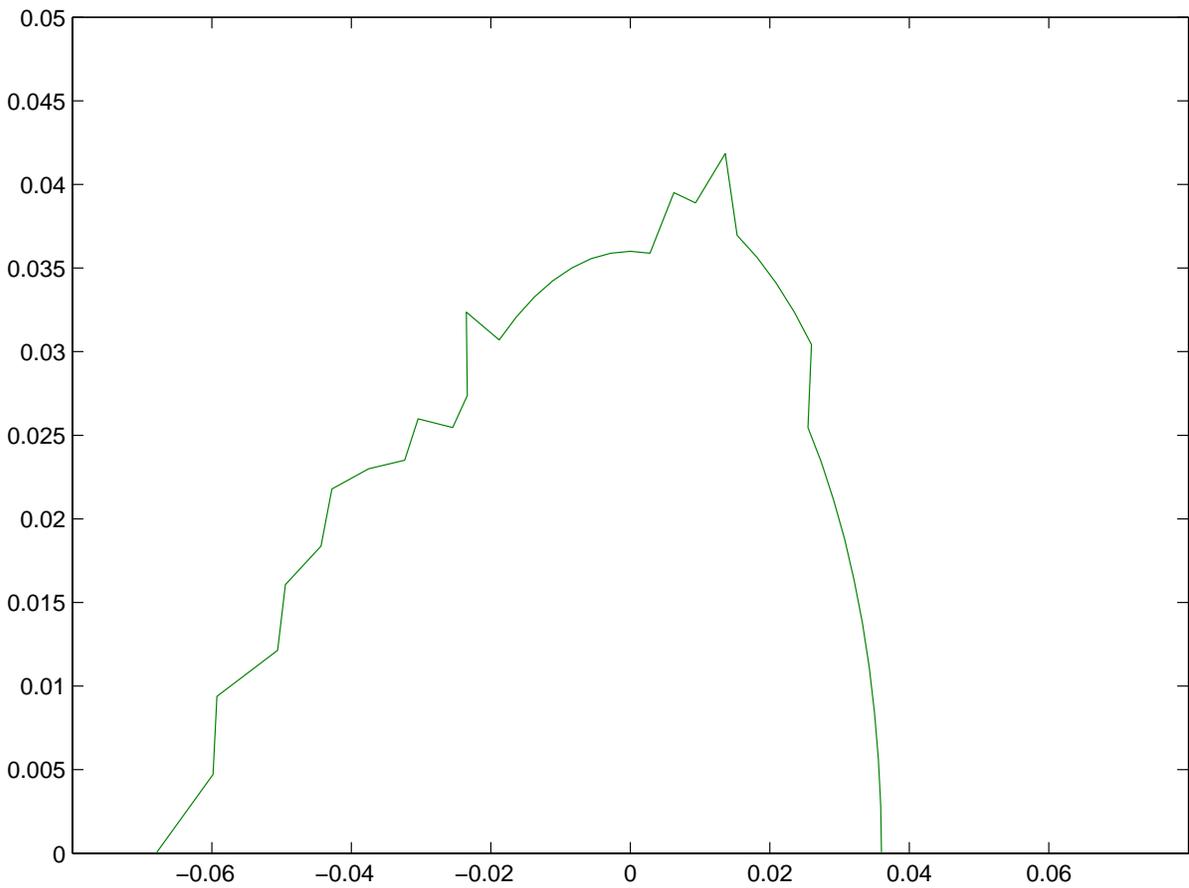


Figure 7

```

OPTION (%%, $, Z, VI, AX, X, DE, MO, FI, DIAG, SY, GR, TR, CO,
        NUDP, XXP, ZZP, LDP, XZ, EXZ, DEFQ, DEFD, DEFM, AL
        AJ, AJ2, SX, SXLD, LDSX, AVAR, NLOG, MO2) : defq
SIGMA      X      [0.0000E+00]: 5e-5
SIGMA      Z      [0.0000E+00]: 5e-5
SIGMA      DG/G   [0.0000E+00]:
SIGMA      S      [0.0000E+00]:
SIGMA      ROT S  [0.0000E+00]:
PREVIOUS SEED NUMBER      A41F1AB3;
ACTUAL SEED NUMBER      [A41F1AB3]:
Number of samples      [ 1]: 10
DP/P      [ 0.00000E+00]:
PRECISION C.O      [ .10000E-06]:
NB STEPS, NB TURNS/STEP [ 200 1]:
ORBIT CORRECTION (N,Y)      :
2ND ORDER RESONANCES (N,Y)  :
PLOT (%%, OTF, COX, COZ, BETX, BETZ, DX, DZ, ENVX, ENVZ, ENVZ, CPL, HARM, HCQ, HSK, HH, VI) :
DYNAMIC APERTURE (N,Y)      :
Max c.o.limit for stat. [ 1.0000]:

```

*** QUADRUPOLE ERRORS ACTIVATED ***

```

1   A41F1AB3   A41F1AB3
7   -3.2220E-04 -2.1814E-05 -1.6763E-03  7.0792E-05 -5.8970E-04  0.0000E+00
STEP 0 INITIAL      : RMS P.U.   2.164E-03           1.967E-03
                    MAX C.O.   5.758E-03           5.072E-03
                    RMS C.O.   2.475E-03           1.756E-03
                    MAX DCOR   0.000E+00           0.000E+00

```

```

NU X,Z ,mom. compact.: 4.45456E-01  3.81093E-01  2.84216E-04

```

```

EMPROJ X,Z coupling : 3.37885E-08  8.05195E-09  2.38304E-01

```

```

EM X,Z coupling : 3.37885E-08  8.05195E-09  2.38304E-01

```

```

SEXT C.O. aver,rms : 8.17429E-06  2.35381E-03

```

```

foc. aver,rms : 5.21884E-05  9.67709E-03

```

```

BETA X,Z min max mod : 1.91221E+01  4.71729E+01  5.96073E-01

```

```

1.20383E+01  1.23304E+01  1.30650E-02

```

DISP X min max rms aver : -5.85582E-01 7.99438E-01 3.81731E-01 -1.20268E-02
DISP Z min max rms aver : -2.46458E-01 2.01585E-01 1.52766E-01 -2.82513E-04
SIZE X min max rms aver : 8.05540E-04 1.26480E-03 1.25305E-04 1.01684E-03
SIZE Z min max rms aver : 3.10356E-04 3.16675E-04 1.65915E-06 3.12942E-04
1 A41F1AB3 A41F1AB3 COMPLETED

2 10332B25 A41F1AB3
... ..
7 -1.4856E-03 -3.2481E-05 -2.4776E-03 1.5619E-04 -1.4398E-03 0.0000E+00
STEP 0 INITIAL : RMS P.U. 3.169E-03 2.244E-03
MAX C.O. 1.031E-02 5.710E-03
RMS C.O. 3.686E-03 2.053E-03
MAX DCOR 0.000E+00 0.000E+00

NU X,Z ,mom. compact.: 4.30180E-01 3.82652E-01 1.64083E-04

EMPROJ X,Z coupling : 8.73385E-08 9.64216E-09 1.10400E-01

EM X,Z coupling : 8.73385E-08 9.64216E-09 1.10400E-01

SEXT C.O. aver,rms : -4.29873E-05 3.44097E-03
foc. aver,rms : 5.71054E-05 1.40847E-02

BETA X,Z min max mod : 2.13767E+01 4.14939E+01 4.03927E-01
1.13447E+01 1.26442E+01 6.99279E-02

DISP X min max rms aver : -1.59074E+00 1.25130E+00 8.14066E-01 -2.04727E-02
DISP Z min max rms aver : -2.63453E-01 2.37475E-01 1.57110E-01 -2.99664E-03
SIZE X min max rms aver : 1.37308E-03 1.91332E-03 1.78320E-04 1.62323E-03
SIZE Z min max rms aver : 3.36873E-04 3.58689E-04 5.96755E-06 3.47791E-04

10 A8A3AD6 A41F1AB3 COMPLETED

STATISTICS FOR 10 MACHINES (0 REJECTED)

0 INITIAL : RMS P.U. 3.301E-03 1.855E-03
MAX C.O. 1.368E-02 6.854E-03
RMS C.O. 3.883E-03 1.668E-03

NU Xave,Xsdv,Zave,Zsdv: 4.12992E-01 1.27022E-02 3.78294E-01 5.19344E-03

```

SEXT   C.O. aver,rms : -5.75757E-05  3.58150E-03
      foc. aver,rms  :  4.40670E-05  1.46609E-02
AUTOMATIC SCALES (Y,N)      :
```

Figure 8

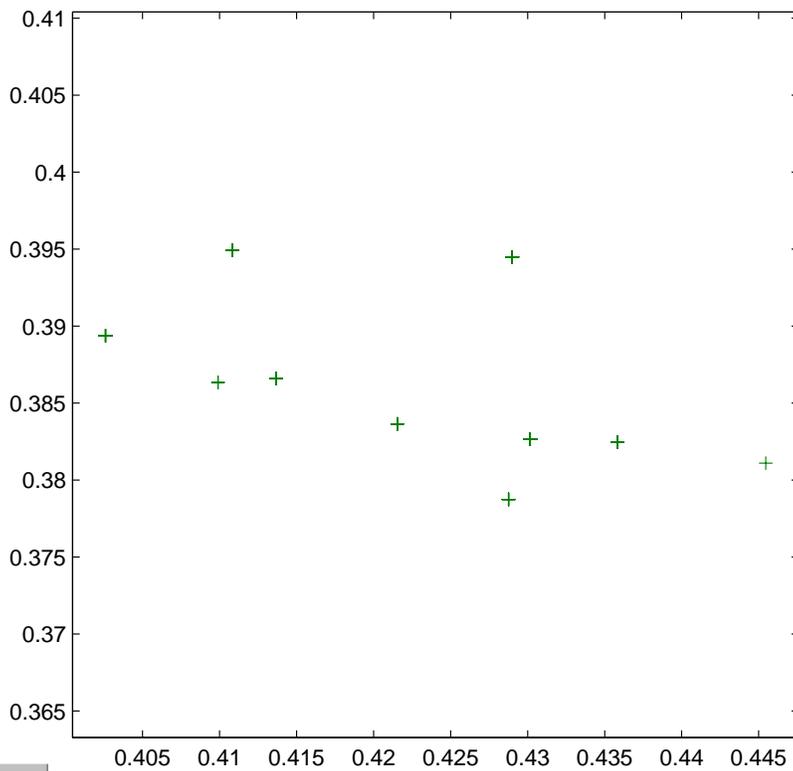
```

DNuX = .41299 <DNuX> = .01270
DNuZ = .37829 <DNuZ> = .00519
```

*** QUADRUPOLES ***

```

< X  > = 5.000E-05
< Z  > = 5.000E-05
```



*** DIPOLES ***

*** MULTIPOLES ***

Stop

Figure 9

```

OPTION(%%, $, Z, VI, AX, X, DE, MO, FI, DIAG, SY, GR, TR, CO,
      NUDP, XXP, ZZP, LDP, XZ, EXZ, DEFQ, DEFD, DEFM, AL
      AJ, AJ2, SX, SXLD, LDSX, AVAR, NLOG, MO2) : aj
PRINT OUT OF THE STRUCTURE (N/Y) :
      *** DATA FOR THE FIT ***
NUMBER OF VARIABLES      [ 2] :
ELEM PARAM COUPL
S13      2
S19      2
NUMBER OF CONSTRAINTS   [ 2] :
TYPE OF CONSTRAINTS: 0=BEAM, 1=MAT, 2=NU, 5=Ex/Gamma**2, 6=DISPERSION
      3=2RD ORDER, 4=ACCEPTANCE
TOTAL NUMBER OF ELEMENTS IN THE STRUCTURE:      139
TYPE   I   J  NUM          VAL          DVAL
   3   1   0  139      1.20000E-01      1.00000E+00
   3   2   0  139      2.50000E-01      1.00000E+00
NUMBER OF VARIABLES      [ 2] : 2
ELEM PARAM COUPL
QD8      2
QF7      2
NUMBER OF CONSTRAINTS   [ 2] : 2
TYPE OF CONSTRAINTS: 0=BEAM, 1=MAT, 2=NU, 5=Ex/Gamma**2, 6=DISPERSION
      3=2RD ORDER, 4=ACCEPTANCE
TYPE   I   J  NUM          VAL          DVAL
   2   1   0  139          0.25          1.
   2   3   0  139          0.65          1.
initial stepsize [1.0000E+00]: 0.1
TYPE ^C TO INTERRUPT

   0          .10000E+21          .45877E-02 *
   1          .45877E-02          .14218E-01 .
   0          .40987E-02          .47553E-02 .
   1          .40987E-02          .39956E-02 *
   2          .39956E-02          .41328E-02 .
  -2          .39956E-02          .45555E-02 .
   ...          ...          ...
   1          .11322E-09          .11322E-09 .
  -1          .11322E-09          .11322E-09 .

```

```

2          .11322E-09          .11322E-09 .
-2         .11322E-09          .11322E-09 .
0          .11322E-09          .11322E-09 .
*** PRESENT STATE OF VARIABLES ***

ELEM NB VAR NB PAR NB   MINIMUM   INI.VALUE   FIN.VALUE   MAXIMUM   STEP
  32    1    2    2   -1.120E+00 -1.63952E-01 -1.63952E-01  1.000E-01  1.299E-11
  33    2    2    2   -1.000E-01  7.31584E-01  7.31584E-01  1.737E+00  5.795E-13

*** PRESENT STATE OF FIT ***

IC  I  J  NOEL    DESIRED VAL    DVAL        REACHED VAL    KI2
  2  1  0  139     2.50000E-01    1.00        2.50003E-01    .104
  2  3  0  139     6.50000E-01    1.00        6.50010E-01    .896

OPTION   : BETA
STRUCTURE: D:\beta\structures\nom7nm_pc.str
0/ 139

                FIRST-ORDER MATRIX

-2.161E-05  2.299E+01  0.000E+00  0.000E+00  0.000E+00 -1.021E-03  0.000E+00
-4.349E-02 -2.161E-05  0.000E+00  0.000E+00  0.000E+00 -4.440E-05  0.000E+00
 0.000E+00  0.000E+00 -5.877E-01 -6.535E+00  0.000E+00  0.000E+00  0.000E+00
 0.000E+00  0.000E+00  1.002E-01 -5.877E-01  0.000E+00  0.000E+00  0.000E+00
-4.440E-05 -1.021E-03  0.000E+00  0.000E+00  1.000E+00  1.496E-02  0.000E+00
 0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00  1.000E+00  0.000E+00

                MACHINE RADIUS          =          8.399307
                CELL LENGTH              =          52.774403

                NUX=      .250003      NUZ=      .650010      NUS=*****

```

Figure 10

OPTION(%%, \$, Z, VI, AX, X, DE, MO, FI, DIAG, SY, GR, TR, CO,

NUDP,XXP,ZZP,LDP,XZ,EXZ,DEFQ,DEFD,DEFM,AL
 AJ,AJ2,SX,SXLD,LDSX,AVAR,NLOG,MO2) : aj2
 PRINT OUT OF THE STRUCTURE (N/Y): N
 *** DATA FOR THE FIT ***
 NUMBER OF VARIABLES [2]:
 ELEM PARAM COUPL
 S13 2
 S19 2
 NUMBER OF CONSTRAINTS [2]:
 TYPE OF CONSTRAINTS: 0=BEAM,1=MAT,2=NU,5=Ex/Gamma**2,6=DISPERSION
 3=2RD ORDER,4=ACCEPTANCE

TOTAL NUMBER OF ELEMENTS IN THE STRUCTURE: 139

I= 1:CSIX , 2:CSIZ , 3:DBETX , 4:DALPX , 5:DBETZ , 6:DALPZ
 7:DNUX/EX , 8:DNUX/EZ , 9:DNUZ/EX , 10:DNUZ/EZ J=NHARM
 11:SOR/EX ,12:SOR/EZ ,13:SOR/EXZ
 14:D ,15:F ,16:G J=NHARM
 17:DG ,18:L ,19:H J=NHARM

I=1,J=1 :ACCEPTANCE X , I=3,J=3 :ACCEPTANCE Z

TYPE	I	J	NUM	VAL	DVAL
3	1	0	139	1.20000E-01	1.00000E+00
3	2	0	139	2.50000E-01	1.00000E+00

NUMBER OF VARIABLES [2]: 4
 ELEM PARAM COUPL
 S4 2
 S6 2
 S22 2
 S24 2
 NUMBER OF CONSTRAINTS [2]: 9
 TYPE OF CONSTRAINTS: 0=BEAM,1=MAT,2=NU,5=Ex/Gamma**2,6=DISPERSION
 3=2RD ORDER,4=ACCEPTANCE

I= 1:CSIX , 2:CSIZ , 3:DBETX , 4:DALPX , 5:DBETZ , 6:DALPZ
 7:DNUX/EX , 8:DNUX/EZ , 9:DNUZ/EX , 10:DNUZ/EZ J=NHARM
 11:SOR/EX ,12:SOR/EZ ,13:SOR/EXZ
 14:D ,15:F ,16:G J=NHARM
 17:DG ,18:L ,19:H J=NHARM

I=1,J=1 :ACCEPTANCE X , I=3,J=3 :ACCEPTANCE Z

TYPE	I	J	NUM	VAL	DVAL
------	---	---	-----	-----	------

3	7	200	0	0	1.
3	8	200	0	0	1.
3	10	200	0	0	1.
3	14	1	0	0	1.
3	15	1	0	0	1.
3	16	1	0	0	1.
3	17	1	0	0	1.
3	18	1	0	0	1.
3	19	1	0	0	1.

*** FIRST ORDER TRANSFER ***

TYPE ^C TO INTERRUPT

0	.10000E+21	.23837E+09 *
1	.23837E+09	.64345E+09 .
...
3	.39624E+08	.40267E+08 .
-3	.39624E+08	.41394E+08 .
4	.39624E+08	.43160E+08 .
-4	.39624E+08	.41212E+08 .
0	.39624E+08	.40416E+08 .
1	.39624E+08	.46752E+08 .
-1	.39624E+08	.51397E+08 .
2	.39624E+08	.39740E+08 .
-2	.39624E+08	.40416E+08 .
3	.39624E+08	.40267E+08 .
-3	.39624E+08	.41394E+08 .
4	.39624E+08	.43160E+08 .
-4	.39624E+08	.41212E+08 .
...
-3	.23603E+08	.23603E+08 .
4	.23603E+08	.23603E+08 .
-4	.23603E+08	.23603E+08 .

*** PRESENT STATE OF VARIABLES ***

ELEM	NB	VAR	NB	PAR	NB	MINIMUM	INI. VALUE	FIN. VALUE	MAXIMUM	STEP
4	1	2				-.318E+08	.1935E+07	.19352E+07	.318E+08	.410E-03
6	2	2				-.314E+08	-.2217E+07	-.22173E+07	.314E+08	.405E-03
22	3	2				-.230E+08	-.1142E+07	-.11422E+07	.230E+08	.297E-03
24	4	2				-.424E+08	.1851E+07	.18511E+07	.424E+08	.164E-02
0						.23603E+08	.23603E+08			

*** PRESENT STATE OF FIT ***

IC	I	J	NBEL	DESIRED VAL	DVAL	REACHED VAL	KI2
3	7	200	0	.0000E+00	.1000E+01	-.1218E+04	.6283E-01

```

3   8 200   0   .0000E+00   .1000E+01   .1601E+04   .1086E+00
3  10 200   0   .0000E+00   .1000E+01   .3489E+04   .5156E+00
3  14   1   0   .0000E+00   .1000E+01   .1941E+04   .1596E+00
3  15   1   0   .0000E+00   .1000E+01   -.1395E+04   .8240E-01
3  16   1   0   .0000E+00   .1000E+01   .4864E+03   .1002E-01
3  17   1   0   .0000E+00   .1000E+01   .7880E+03   .2631E-01
3  18   1   0   .0000E+00   .1000E+01   -.4035E+03   .6897E-02
3  19   1   0   .0000E+00   .1000E+01   -.8089E+03   .2772E-01

```

OPTION : BETA

STRUCTURE: D:\beta\structures\nom7nm_pc.str

0/ 139

FIRST-ORDER MATRIX

```

-1.719E-01  2.912E+01  0.000E+00  0.000E+00  0.000E+00 -1.324E-03  0.000E+00
-3.333E-02 -1.719E-01  0.000E+00  0.000E+00  0.000E+00 -3.767E-05  0.000E+00
 0.000E+00  0.000E+00 -2.371E-01 -1.185E+01  0.000E+00  0.000E+00  0.000E+00
 0.000E+00  0.000E+00  7.965E-02 -2.371E-01  0.000E+00  0.000E+00  0.000E+00
-3.767E-05 -1.324E-03  0.000E+00  0.000E+00  1.000E+00  1.496E-02  0.000E+00
 0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00  1.000E+00  0.000E+00

```

```

MACHINE RADIUS      =      134.388916
CELL LENGTH         =      52.774403

```

```

NUX=   .439969      NUZ=   .390390      NUS=*****

```

Figure 11

The last 3 options are executed over 1 superperiod :

```

OPTION (%%, $, Z, VI, AX, X, DE, MO, FI, DIAG, SY, GR, TR, CO,
        NUDP, XXP, ZZP, LDP, XZ, EXZ, DEFQ, DEFD, DEFM, AL
        AJ, AJ2, SX, SXLD, LDSX, AVAR, NLOG, MO2) : sx
PLOT (SE, HA, %%) : se
NSEX: 14
MAX. VERT. SCALE [ .50000E+02]: 25.
SCALING FACTOR [ .10000E+07]:
RESONANCE M*NUX+N*NUZ=P M,N,P[ 0 0 0]: 1 2 4
AXIS (Y,N) : y

```

1	.93170E+01	.11695E+02
2	-.58937E+01	-.19590E+02
3	-.27576E+00	.16009E+02
4	.77417E+01	-.21598E+02
5	-.10070E+02	.12716E+02
6	.78500E+01	-.54276E+01
7	-.68295E+01	.41589E+00
8	-.68295E+01	-.41592E+00
9	.78500E+01	.54276E+01
10	-.98759E+01	-.12471E+02
11	.77417E+01	.21598E+02
12	-.27572E+00	-.16009E+02
13	-.58937E+01	.19590E+02
14	.93170E+01	-.11695E+02

PLOT (SE, HA, %%) :

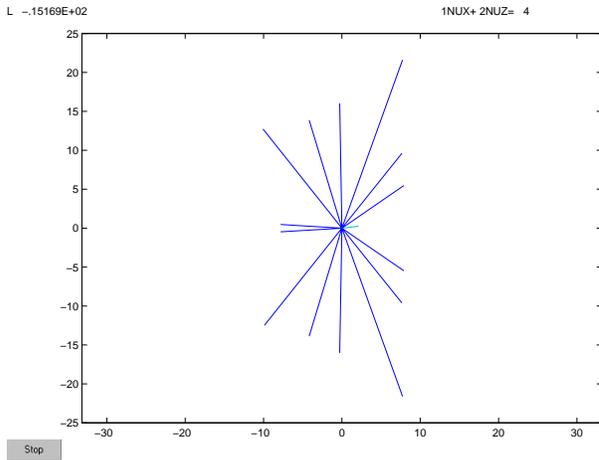


Figure 12

```

OPTION (%%, $, Z, VI, AX, X, DE, MO, FI, DIAG, SY, GR, TR, CO,
        NUDP, XXP, ZZP, LDP, XZ, EXZ, DEFQ, DEFD, DEFM, AL

```

```

AJ,AJ2,SX,SXLD,LDSX,AVAR,NLOG,MO2) : sx
PLOT(SE,HA,%%) : ha
PLOT (D,F,G,H,L,DG) : f
NB OF HARMONICS [ 200]:
HOR. SCALE MIN,MAX [-9.30000E+01 1.06000E+02]: -100 100
MAX. VERT. SCALE [ 1.00000E+04]: 5000.
PLOT(SE,HA,%%) :

```

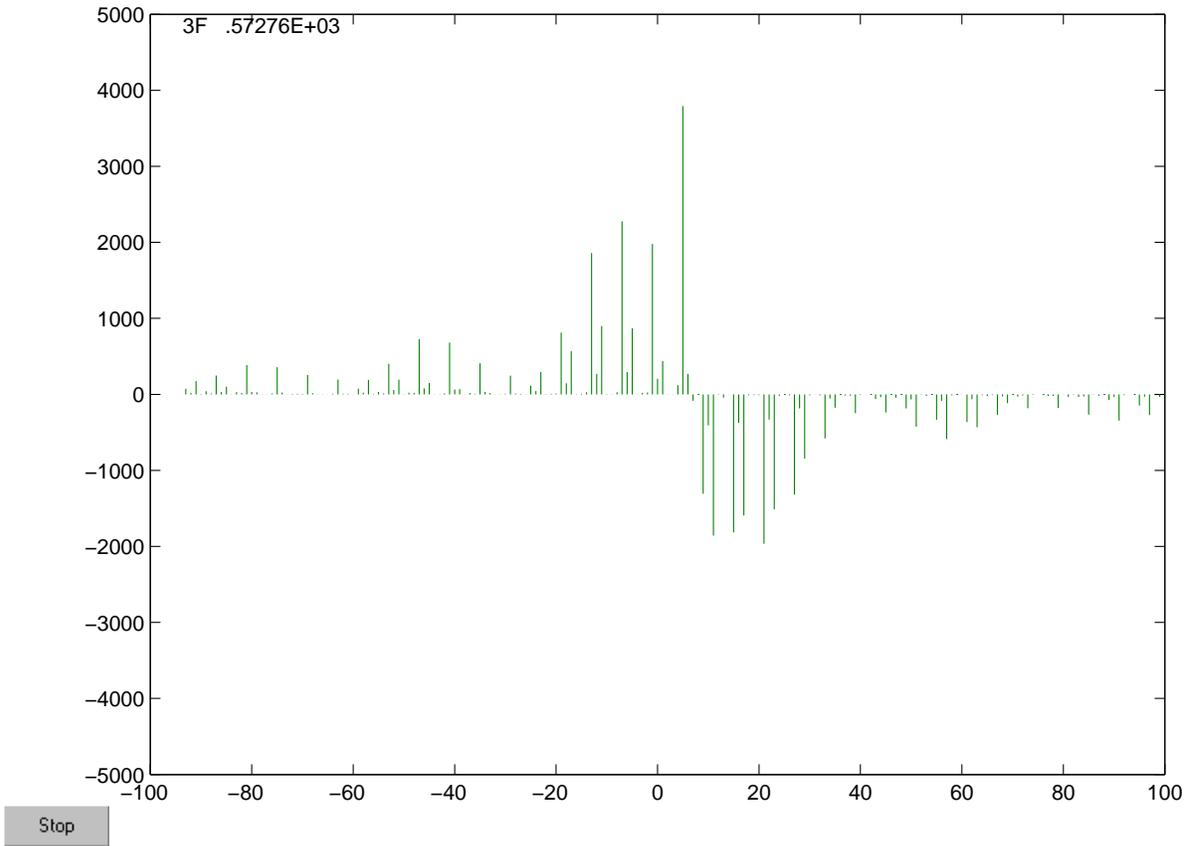


Figure 13

```

OPTION(%%,$,Z,VI,AX,X,DE,MO,FI,DIAG,SY,GR,TR,CO,
NUDP,XXP,ZZP,LDP,XZ,EXZ,DEFQ,DEFD,DEFM,AL

```

```

AJ,AJ2,SX,SXLD,LDSX,AVAR,NLOG,MO2) : diag
NUX MIN,MAX [ .0000 .5000]: 1.9 2.6
NUZ MIN,MAX [ .0000 .5000]: 0.4 1.1
RESONANCE LINES, TUNE FILE, EXIT (RE,TR,%%): RE
TYPE OF RESONANCE (SY,RA,AL) : SY
REGULAR,SKEW,ALL (RE,SK,AL) : RE
ORDER [ 2]: 3
MAX HARMONIC [ 500]: 500
RESONANCE LINES, TUNE FILE, EXIT (RE,TR,%%): TR

```

```

2.2 .7625
2.2 .6375
2.2625 .7625
2.2625 .6375

```

[EOF]

```

RESONANCE LINES, TUNE FILE, EXIT (RE,TR,%%): RE
TYPE OF RESONANCE (SY,RA,AL) : SY
REGULAR,SKEW,ALL (RE,SK,AL) : RE
ORDER [ 3]: 5
MAX HARMONIC [ 500]:
RESONANCE LINES, TUNE FILE, EXIT (RE,TR,%%): %%

```

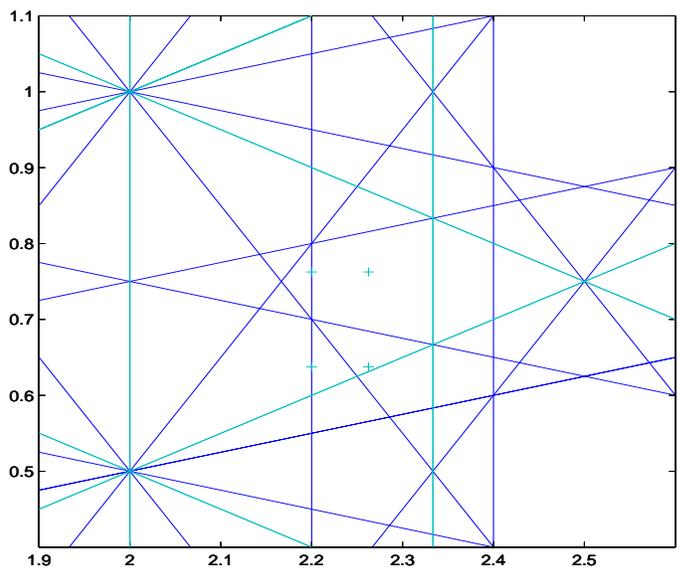


Figure 14

APPENDIX C : LONGITUDINAL MOTION

The proper use of the longitudinal tracking option **LDP** is somewhat tricky, especially for low emittance machines where the momentum compaction factor is very small and non-linear contributions become important.

In the following we make an attempt to describe the logical step for the running option. Unfortunately, the procedure is not completely automatic and requires some care from the user.

- Determination of the 6 coordinates of the closed orbit by using **CO** $x_{co}, x'_{co}, z_{co}, z'_{co}, l_{co}, \delta_{co}$. If the initial transverse coordinates are zero, the use of **LDP** is straightforward. However, in case of non-zero initial transverse coordinates, the average path length depends quadratically on the energy and one has to determine the point with no path length variation in l-dp space.
- **LDP** is run without cavities and no element simulating an energy loss or an acceleration. Particles are tracked with the desired transverse coordinates and different energy displacements to find the δ where the path length variation is zero. The figure shows the value δ_{amp} for which a particle remains synchronous. It has to be noted that, for large transverse amplitudes, the path length varies from turn to turn and one has to deal with only an approximately synchronous particle.

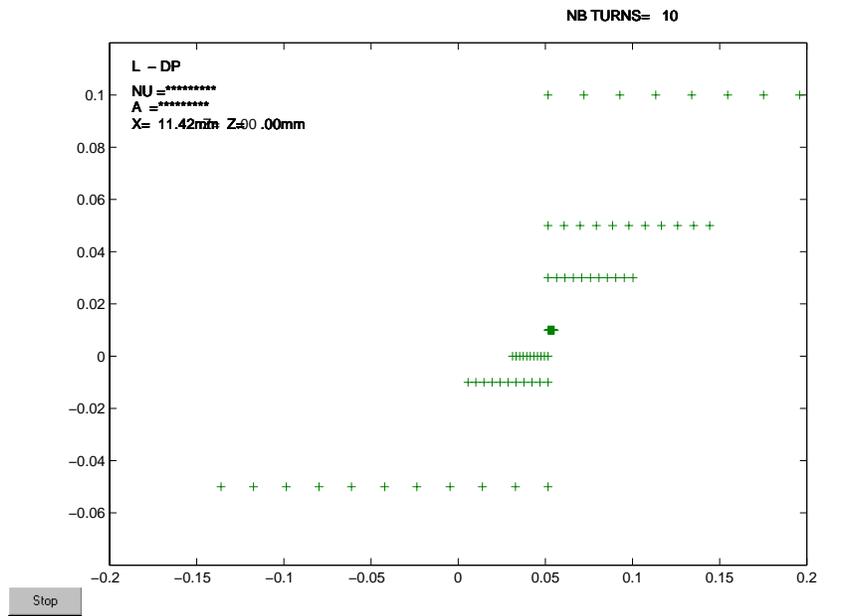


Figure 1

- We switch on the cavities and the energy displacing elements and carry out with them the real tracking. Running **LDP** requires the following input:

```

OPTION (%%, $, Z, VI, AX, X, DE, MO, FI, DIAG, SY, GR, TR, CO,
        NU DP, XXP, ZXP, LDP, XZ, EXZ, DEFQ, DEFD, DEFM, AL
        AJ, AJ2, SX, SXLD, LDSX, AVAR, NLOG, MO2) :ldp
****      CHROMATIC CLOSED ORBIT      ****
DP/P      [      .00000E+00]:  $\delta_{amp}$  + desired longitudinal amplitude
OPTION (%%, EP, YYP, NP, DE) :de
XC       :                .11840E-02
X'C      :                -.20422E-07
ZC       :                .00000E+00
Z'C      :                .00000E+00
LC       :                .25441E-03
DP/PC    :                .20000E-01

XC       .11840E-02 : unchanged
X'C      -.20422E-07 : unchanged
ZC       .00000E+00 : unchanged
Z'C      .00000E+00 : unchanged
LC       .25441E-03 :  $l_{co}$  of step 1
DP/PC    .20000E-01 : unchanged
*****NON-ZERO CLOSED ORBIT*****
ONE PARTICLE  EX, PHX, EZ, PHZ, ES, PHS..... :EP
ONE PARTICLE  X, X', Z, Z', L, DP/P..... :YYP
NX*NZ*NS PARTICLES EX, PHX, EZ, PHZ..... :NP
C.O COORDINATES XC, X'C, ZC, Z'C, LC, DP/P.. :DE
OPTION (%%, EP, YYP, NP, DE) :yyp
...
PREVIOUS INITIAL OR FINAL CONDITIONS (IN OR FI):IN
X          .00000E+00 : desired transverse amplitudes
X'         .00000E+00 :  "
Z          .00000E+00 :  "
Z'         .00000E+00 :  "
L          .00000E+00 : 0
DP/P       .00000E+00 : 0

```

Figure 2

APPENDIX D : EMITTANCES

For electron machines, BETA computes the damping coefficients and equilibrium emittances by two different methods:

1) No horizontal-vertical-longitudinal coupling [1]:

The standard synchrotron radiation integrals are cumulated in each dipole:

$$I_1 = \oint \left(\frac{\eta}{\rho} \right) ds \quad I_2 = \oint \left(\frac{1}{\rho} \right)^2 ds \quad I_3 = \oint \left| \frac{1}{\rho} \right|^3 ds$$

$$I_4 = \oint \frac{(1 - 2\eta)}{\rho^3} ds \quad I_5 = \oint \frac{(\gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2)}{|\rho|^3} ds$$

The damping partition numbers are then computed:

$$J_x = 1 - (I_4/I_2) \quad J_z = 1 \quad J_\epsilon = 2 + (I_4/I_2)$$

The average radiated power is :

$$P_\gamma = C_\gamma \cdot \frac{E^4}{2\pi T_0} I_2 \quad \text{with } C_\gamma = 8.85 \cdot 10^{-5} \text{ m (GeV)}^{-3}$$

The damping times are deduced by:

$$\tau_i = \frac{2 E}{J_i \cdot P_\gamma}$$

and the horizontal emittance is:

$$\varepsilon_x = C_q \frac{\gamma^2}{J_x} \frac{I_5}{I_2} \quad \text{with } C_q = 3.84 \cdot 10^{-13} \text{ m}$$

2) General method [2]:

The eigen values $e^{j\phi_i}$ and eigen vectors \mathbf{V}_i of the 6x6 transfer matrix of one superperiod are computed:

$$\mathbf{V}_i = \begin{pmatrix} V_{1i} \\ V_{2i} \\ V_{3i} \\ V_{4i} \\ V_{5i} \\ V_{6i} \end{pmatrix}, \text{ with } \mathbf{V}_2 = \mathbf{V}_1^*, \mathbf{V}_4 = \mathbf{V}_3^*, \mathbf{V}_6 = \mathbf{V}_5^* \text{ and } \phi_2 = -\phi_1, \phi_4 = -\phi_3, \phi_6 = -\phi_5$$

ϕ_1, ϕ_3, ϕ_5 give the tunes of the coupled machine. The position of a particle in the 6-dimensional phase-space $(x, x', z, z', l, \delta)$ can be expressed by its coordinates in the basis of the eigen-vectors:

$$\mathbf{X} = \begin{pmatrix} x \\ x' \\ z \\ z' \\ l \\ \delta \end{pmatrix} = (\mathbf{V}) \begin{pmatrix} A_1 \\ A_1^* \\ A_3 \\ A_3^* \\ A_5 \\ A_5^* \end{pmatrix} \quad \text{Where the } A_i \text{'s are constant along the particle trajectory.}$$

The three families of particles

$$|A_1| = \text{constant}, |A_3| = 0, |A_5| = 0$$

$$|A_1| = 0, |A_3| = \text{constant}, |A_5| = 0$$

$$|A_1| = 0, |A_3| = 0, |A_5| = \text{constant}$$

are such that they will remain identical after one superperiod. They can then define 3 independent matched beams. With an adequate normalisation of the eigen-vectors, BETA displays the 3 corresponding beam matrices σ_{ijk} $k=1,3,5$

$$\text{such that } \tilde{X} \sigma_k^{-1} X = \varepsilon_k$$

When there is no coupling, these 3 matrices are reduced to:

$$\sigma_1 = \begin{pmatrix} \beta_x & & & & & \\ -\alpha_x & \gamma_x & & & & \\ 0 & 0 & 0 & & & \\ 0 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 0 & & & & & \\ 0 & 0 & & & & \\ 0 & 0 & \beta_z & & & \\ 0 & 0 & -\alpha_z & \gamma_z & & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \sigma_5 = \begin{pmatrix} 0 & & & & & \\ 0 & 0 & & & & \\ 0 & 0 & 0 & & & \\ 0 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & \beta_s & \\ 0 & 0 & 0 & 0 & -\alpha_s & \gamma_s \end{pmatrix}$$

The actual beam matrix giving the envelopes around the machine is $\Sigma = \varepsilon_1 \sigma_1 + \varepsilon_3 \sigma_3 + \varepsilon_5 \sigma_5$. At any point, beam envelopes are given by $\hat{x} = \sqrt{\Sigma_{11}}$, $\hat{z} = \sqrt{\Sigma_{33}}$, $\hat{l} = \sqrt{\Sigma_{55}}$

For proton machines, the 3 constants ε_1 , ε_3 , ε_5 have to be introduced by the operator before plotting envelopes. For electron machines the equilibrium emittances are computed by looking at the fluctuation of the constants A_i due to radiation in the dipoles, and acceleration in the cavities. This gives 3 equilibrium invariants corresponding to the 3 beam matrices which describe in the phase space a six-dimensional ellipsoid. The horizontal and vertical emittances are then given either the projections of the six-dimensional ellipsoid on vertical and horizontal plane or cross-sections of the ellipsoid with the plane $\delta = 0$. The coupling is defined as the ratio between vertical and horizontal monochromatic emittances. It should be noticed however, that the emittances are computed at the initial point, but may vary (but not the invariants !) along the machine.

If there is no cavity and no energy displacing element in the structure list, this computation is applied with the 4x4 formalism, assuming that a cavity restores exactly the energy loss due to radiation. Horizontal and vertical emittances are then computed. If there is a cavity in the structure, it is necessary to introduce one or several energy displacements simulating the radiation losses, and then horizontal, vertical and longitudinal emittances are computed. For a machine without coupling, all results are in perfect agreement.

3) Emittance Output:

The values of the quantities explained in the previous section are displayed within the first analysis of the structure. In the first analysis the section headed by "DAMPING TIMES" shows all emittance values. It displays the horizontal emittance as it is given by the radiation integrals. In a second part the quantities related to the eigenvector method are shown. In the first column of this part the damping times are displayed, the second column is empty, the third column headed by "INVARIANT" displays the invariants ε_1 and ε_3 , the fourth column headed by "EMITTANCE PROJECTION" displays the projections of the six-dimensional ellipsoid on the horizontal and the vertical planes, and in the last column headed by "DP/P=0" the cross-sections of the phase space ellipsoid with the plane $\delta=0$ are displayed finally followed by a line named "COUPLING" which displays the ratio of all the values indicated in the two preceding lines representing the coupling. In case of a only four-dimensional movement of the particles (no cavity in the structure) the values obtained by projection or cross-section are identical and represent what is usually understood as emittances. The possible variation of the coupling obtained by emittances by projection can be displayed within **DEFQ** using the graphics option CPL. All emittance values are indicated in units of meter.

REFERENCES

- [1] R.H.Helm & al., IEEE trans. Nucl. Sci. 20, p.900 (1973)
- [2] J.P. Bardin: Anneau de stockage à électrons multiplicateur de fréquence. Thesis, Université de Paris-sud, Orsay (1973)

APPENDIX E : MAPPED INSERTION DEVICES

Mapped insertion devices are based on the description of insertion devices by interpolation tables which provide a kick as a function of the coordinates of the particle passing the element. This description is more general and can be more precise than the analytical description by a mathematical formula.

As the use of this element in BETA has been already explained, this appendix will only be devoted to the table format and the physical principles on which mapped insertion devices are based on.

Table format

The following lines show the table format to be used for mapped insertion devices. Everything in parentheses as well as arrows are explanations to the format and does not belong to the format itself.

```
# comment
.
# comment
L          (length of the ID)
# comment
.
.
.
# comment
NL          (number of horizontal points, maximal value 200)
# comment
.
.
# comment
NC          (number of vertical points, maximal value 50)
# comment
.
.
# comment
START (-----> increasing x-coordinates)
      x1   x2   x3   .....  x(NL)   | (table of horizontal kicks in T2m2)
z1    a11  a12  a13  .....  a1NL   |
z2    a21  a22  a23  .....  a2NL   | (decreasing z-coordinates)
.     .   .   .   .   .   .   |
.     .   .   .   .   .   .   |
z(NC) aNC1 aNC2 aNC3 ..... aNCNL  V
# comment
.
.
# comment
START (-----> increasing x-coordinates)
```

	x1	x2	x3	x(NL)		(table of vertical kicks in T ² m ²)
z1	b11	b12	b13	b1NL		
z2	b21	b22	b23	b2NL		(decreasing z-coordinates)
.		
.		
z(NC)	bNC1	bNC2	bNC3	bNCNL	∨	

comment

End of the table. The keyword START is absolutely necessary to allow BETA to recognize the beginning of the table. The coordinates at the borders of the tables are only used to calculate the step width (step horizontal = (X(NL)-X1)/(NL-1), step vertical = (Z(NC)-Z1)/(NC-1)). Concerning the tables maximal 200 values are allowed in horizontal direction, maximal 50 values are allowed in vertical direction (This can of course be easily changed by an intervention of the code, recompilation necessary).

The program checks the input on consistency and if any inconsistency is found an error message is issued.

The element in BETA is treated as several thin lenses separated by spaces which correspond to the total length of the device divided by the number of iterations. In the case of only one iteration no spaces exist apart from the drift spaces before and after the "thin lens". The device is set up so that it does not occupy length in the structure, i.e. the drift spaces at both sides of the device do not have to be changed in case of an installation.

In the graphics the mapped insertion device can be distinguished from a standard one by the diagonal which is plotted additionally in the rectangle symbolising normally an insertion device.

The use of mapped insertion devices also leads to a limitation in the physical aperture. Due to the fact that the map describing the insertion device has a finite size, a particle whose coordinates are outside the map cannot be tracked any longer. It is lost. The tracking in BETA takes that into account and a corresponding message is issued. This effect cannot be neglected. Simulations clearly show that this effect has an impact on the dynamical aperture [1].

Physical principle

It is supposed that an insertion device in a machine is optimized so that the beam does not suffer from remaining field integrals. In this case only the second order field effect enters in the description of the mapped insertion device [2].

$$x' = -\frac{\alpha^2}{2} \int_{-\infty}^{\infty} \frac{\partial}{\partial x} \Phi(x, z, s) ds + o(\alpha^3)$$

$$z' = -\frac{\alpha^2}{2} \int_{-\infty}^{\infty} \frac{\partial}{\partial z} \Phi(x, z, s) ds + o(\alpha^3)$$

with

$$\Phi(x, z, s) = \left(\int_{-\infty}^s B_x ds \right)^2 + \left(\int_{-\infty}^s B_z ds \right)^2$$

x' and z' represent the kicks the particle obtains passing the device. If the integral of Φ along s is known the calculation of x' and z' is very easy. Normally Φ can be obtained from magnetic field measurements of the concerned insertion device, in case of an analytical description of the insertion device Φ can also be

given by an analytical formula.

The factor α is defined in the following way: $\alpha = \frac{c}{p} \cdot scale$, where *scale*, the scaling factor, is the third parameter describing the mapped insertion device in the element. From the formulas above it can be immediately concluded that the scaling with *scale* is not linear, but quadratical. More details about the physical principles leading to the description of mapped insertion devices are given in [2].

REFERENCES

- [1] J. Chavanne, P. Van Vaerenbergh, P. Elleaume, T. Günzel, Recent Achievements and Future Prospect of ID Activities at the ESRF, EPAC2000, Vienna
- [2] P. Elleaume: A new Approach to the Electron Beam Dynamics in Undulators and Wigglers, EPAC92, Berlin

BETA INSTALLATION GUIDE

This chapter has the purpose to give the user some hints about the installation of BETA on his system. BETA can be run mainly on two systems:

- UNIX
- PC

However, versions for others systems can eventually be built also in case of request.

UNIX-Version

The UNIX-version is distributed including the source code, in case of PC only the executable is available. The package is available on the ftp-site of the ESRF (<ftp.esrf.fr>) in form of a compressed tar-file. The tar-file contains the top-directory `betadis` with 6 sub-directories:

- `betaf`: It contains the main code.
- `gksf`: It contains the routines which interface BETA to the GKS-library.
- `psldf`: It contains routines for input/output and some utility routines.
- `minidis`: contains the minigraf-library (graphics running on a X-term).
- `dbeta`: contains some structure files, in particular "betait.str"
- `tables`: contains the tables of the insertion device parametrisations

Each subdirectory contains one (or two) Makefiles. The Makefiles in the directory `gksf` and `psldf` will create each a library which will be linked to the main BETA-routines by starting a Makefile in the directory `betaf`. There are two Makefiles in `betaf` `Makefile_gks` and `Makefile_mini`. They have to be used alternatively for GKS or MINIGRAF.

Use of GKS

In the case you choose GKS as graphics package the GKS-library must reside somewhere on your system. In the Makefile in `betaf` the GKS-library is referred to as `"/usr/local/gksbx/gksbx.a"` You have to change this path corresponding to the place of the GKS-library on your system. Furthermore, you need X11 to get it running. If you do not have GKS on X11 running major changes in the files `OPNxxx` of the directory `gksf` might be necessary, especially files containing routines to open different workstation types. In particular it is probably necessary to write an own routine to open the appropriate workstation³ on your system. Several routines opening different types of workstations are already available in the `gksf`-directory. You have to assure that your own routines are well adapted to your system. Furthermore, you have to take care that your own routines are called in `TRINIT` instead of those already provided by the distribution.

In order to benefit correctly from X11 in routine `OPNX11` the routine `GIN811` is called. In fact this routine is not absolutely necessary, however, its absence can reduce the performance of the graphics window. In particular without the use of `GIN811` the "request input functions" might work differently or not at all. Additionally, the functions provided by `GSCHM`, `GPREC` and `GINCH` have probably to be modified if `GIN811` is not used. The user should refer to the GKS documentation.

The directories `gksf` and `betaf` contain 3 different "header files" `*.h`. They are system dependent.

- 1) `SYSTEM.h` It contains character variable definitions which contain commands carrying out actions on graphics of BETA. The user has to insert his own commands corresponding to the system he is using. These commands correspond to graphic options which can be invoked by using the command `VI`. X11 provides a small additional window beside the graphics window with a list of different graphical options.

³ terminus technicus of GKS, it means the display for the GKS-graphics

- 2) `OPTIONS.h` The 4 options in this header file correspond to commands given in `SYSTEM.h`. The association of the option to the corresponding command in `SYSTEM.h` should be evident.
- 3) `PARPLT.h` This file indicates the line and marker types in BETA. The file is used at all places where lines or markers are chosen for drawing. The user can assign to each line or marker type the in GKS available characteristics he wants. `PARPLT.h` is used in files of the `gksf-` and the `betaf-` directory. In the latter case a direct setting of the line or marker type is necessary. However, in MINIGRAF only one marker type is available.

To take into account differences of system commands used in BETA in the routine `BLOCK_EDIT.f` character variables containing these commands are initialised. The user has to change these initialisations corresponding to his system. In case of UNIX no change is necessary.

To have access to the structure files in the directory `dbeta` the environment variable `DBETA` has to be defined as

```
export DBETA=xxxx/dbeta
```

However, the form of this command depends on the shell used.

Use of MINIGRAF

If you use MINIGRAF instead of GKS all necessary routines are in the directory `minidis`. Two Makefiles are provided to create two libraries:

- 1) the MINIGRAF-library : `libminigraf.a` command: `make -f Makelibminigraf`
- 2) the interface of BETA to MINIGRAF: `libinterface.a` command: `make -f Makelibinterface`

For an installation on a new system inside the Makefiles the macro variable "dir" has to be changed to the directory name where BETA is supposed to be installed. In using the Makefile `Makefile_mini` in the directory `betaf` these libraries are linked with the main code of BETA.

As well as in the case of GKS the directory `minidis` contains 2 "header-files" which are system-dependent:

- 1) `FICHER3.H` contains the system command for the execution of a shell command. It has to be changed in function of the used shell and system.
- 2) `PRINTS.H` It contains the commands to print a postscript graph on different printers. The commands have to be adapted to the used system.

It should be kept in mind that the graphics of MINIGRAF⁴ is not perfect. It contains some bugs. MINIGRAF is a graphics which provides a minimal solution if no other graphics is available. GKS or MATLAB should be used as a more powerful graphics. The use of MATLAB is described in section about the installation of BETA on a PC.

PC-Version (MATLAB Version)

The graphics of the PC-version is based on MATLAB. To interface BETA easily to MATLAB, the code was converted to C. Thus the code became more complicated and therefore it is not distributed for this version. A similar version of BETA using MATLAB is also existing on UNIX. The installation and start of the MATLAB-version is mainly the same on PC as on UNIX. The code is distributed as a zip-file (or as a compressed tar-file for UNIX). It can be easily uncompressed by WINZIP (or `uncompress` and `tar -xvf` for UNIX).

Apart from the dll-routine `betapc.dll`, some additional MATLAB-files are provided in the distribution. They are necessary to interface BETA to MATLAB. Here there are:

1. `betafig.m` opens the window for the graphics
2. `betamessage.m` is called if particular errors occur

⁴ The MINIGRAF version used in the ESRF-distribution is not exactly the same than the SACLAY one. Check eventually which is the most appropriate one.

3. `betaout.m` provides the functionality of the option **VI** in MATLAB
4. `betaaxe.m` provides the correct axes for the graphics
5. `betac.m` simply serves as help file if you type `help betac`

Furthermore, some ASCII-files are provided in the distribution:

1. `it-doc.text` a description of the ID-mapping method
2. `betait.str` a structure file containing a mapped insertion device as element
3. `HU88.txt` a table describing a mapped insertion device

The m-files have to be in the same directory as the `betapc.dll` (`betac.mexhp7` for UNIX). Some settings have to be applied before BETA can be launched. Once MATLAB has started in the menu "File" the option "set path..." has to be selected. A dialog box will pop up and you have to add the pathname of the directory where you placed the `betapc.dll` file to the already existing path list of MATLAB. It has to be checked if the path name has been taken correctly into account. Otherwise the action "set path..." has to be repeated. Then the dialog boxes can be closed. The other possibility to inform MATLAB about the existence of the BETA executable is to type the line command:

```
path(path, '...')
```

where the ... represent the path of the directory in which the executable of BETA resides. To finally start BETA type "betapc" (PC) or "betac" (UNIX) at the MATLAB prompt. BETA will start popping up an empty graphics window. Additionally the normal BETA "look and feel" with the BETA prompt will appear. You are now able to enter commands as described in the manual.

Initialization of the ID-mapping

Concerning the ID-mapping the structure contained in "`betait.str`" uses the table `HU88.txt` in the directory `tables` (or in another directory in case of a MATLAB version). It is reminded that the correct path of the file containing the table has to be indicated. Of course this is also valid for the already prepared table `HU88.txt`.

