Coherent and Incoherent Tune Shift Induced by the Nonlinear Flat-Chamber Resistive-Wall Wake Field with an Application to the LHC Collimator Experiment in the SPS

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Abstract

We derive formulae for the coherent and incoherent tune shift due to the nonlinear resistive-wall wake field for a single beam travelling between two parallel plates. The theoretical predictions are compared with measurements performed on an LHC prototype collimator during the 2004 SPS run. We show that the tune shift derived from the nonlinear wake field yields a better agreement with the measurements than two conventional tune-shift expressions which consider only the linear wake field. In particular, we demonstrate that the nonlinear terms of the resistive-wall wake field become important, if, as in the SPS experiment, the collimator gap is comparable to the rms beam size. In addition, for small gap sizes, the incoherent tune spread induced by the nonlinear wake field increases beam stability via enhanced Landau damping.
1 Introduction

In October 2004, a prototype LHC collimator was installed in the SPS. The tune shift induced by this collimator was measured as a function of the collimator gap size. For small gaps, the measured tune shift was almost two times smaller than the one expected from the conventional theories of resistive-wall dipole wake fields [1]. In this report, we demonstrate that the nonlinear wake field components can explain the difference.

The paper is structured as follows. Section 2 derives a formula for the coherent tune shift starting from the general nonlinear resistive-wall wake field for a single bunch travelling between two parallel plates. This section also includes a numerical example and a discussion of the incoherent tune spread induced by the wake field. In Section 3 we compare the tune-shift predictions from the nonlinear wake-field calculation with those from two linear wake field models — due to Burov-Lebedev and Chao, respectively —, including error estimates, and we compare predictions from all three with the experimental data of the 2004 SPS machine study. In Section 4 we propose a more complete formula, which combines the nonlinear resistive-wall wake field of Piwinski with the time dependence of Burov and Lebedev. The predictions obtained from this general hybrid formula are shown to be in good agreement with the experiment. Section 5 draws some conclusions.

For completeness, in Appendix A we summarize the resistive-wall theory of Burov and Lebedev, which does not take into account higher order terms of the wake field, but instead includes the effect of the finite thickness of the collimator jaws. Appendix B reviews the tune shift expected from the classical resistive-wall theory. Lastly, Appendix C compiles the relevant beam parameters recorded during the SPS experiment, their variation in time, and pertinent error estimates.

2 Nonlinear Wake Field and Tune Shift

We consider a beam travelling between two infinitely wide metallic plates, separated by a narrow vertical gap. This system models the configuration of a vertical collimator. We assume the model of a vertical collimator, since this is the same geometry as the one treated by Piwinski, and also since in many applications, e.g., for linear colliders, the vertical wake fields are the more critical ones.

However, in 2004 the actual experiment with a proton bunch at the SPS was performed using a horizontal collimator. When applying the theory to the experimental conditions at the SPS, later in this report, we, therefore, interchange all horizontal and vertical beam sizes and beta functions, before applying the formulae. In other words, numerical values quoted for the horizontal beam size or beta function were vertical values in the real experiment. Likewise, in reality the horizontal tune shift was measured instead of the vertical one which we compute. This exchange is possible, since the theory is symmetric with respect to the two transverse planes.

2.1 Wake Potential

The nonlinear potential, up to infinite order in the transverse positions of both drive and probe particles, for the resistive-wall wake of a Gaussian bunch passing between two parallel plates was
derived by Piwinski [2] and re-written by Bane, Irwin and Raubenheimer [3]. At location \( \tau = z/\sigma_z \) along the bunch (with \( z > 0 \) denoting positions in front of the bunch center), it is given by

\[
V(x, y, x_0, y_0, \tau) = \kappa f_R(\tau) \Phi(x, y, x_0, y_0) ,
\]

where \((x, y)\) denote the transverse coordinates of the test charge, \((x_0, y_0)\) those of the driving charge, \(\Phi\) the transverse potential function

\[
\Phi(x, y, x_0, y_0) = -\left[ \frac{-x_- \sinh x_+ + y_+ \sin y_+}{\cosh x_- + \cos y_+} + \frac{x_- \sinh x_- + y_- \sin y_-}{\cosh x_- - \cos y_-} \right] ,
\]

with

\[
y_+ = \frac{\pi}{2b}(y + y_0), \quad y_- = \frac{\pi}{2b}(y - y_0), \quad x_- = \frac{\pi}{2b}(x - x_0) ,
\]

and \(2b\) the full vertical gap.

Equation (1) conveniently factorizes the dependence on \(\tau\) and on the transverse coordinates. The coefficient \(\kappa\) and the \(\tau\)-dependent function \(f_R\) are

\[
\kappa = \frac{1}{2} \frac{N_b r_p L}{\gamma \sigma_z} \sqrt{\lambda \sigma_z} \quad \text{(4)}
\]

\[
f_R(\tau) = \frac{\sqrt{2}}{\pi} \int_0^\infty \frac{d\tau'}{\sqrt{\tau'}} e^{-\frac{(\tau + \tau')^2}{2}} ,
\]

\[
= \begin{cases} 
\frac{1}{\sqrt{2}} e^{-\frac{\tau^2}{2}} \sqrt{-\tau} \left( I_{-1/4} \left( \frac{\tau^2}{4} \right) + I_{1/4} \left( \frac{\tau^2}{4} \right) \right) & \text{for } \tau < 0 , \\
\frac{1}{\sqrt{2}} e^{-\frac{\tau^2}{2}} \sqrt{\tau} K_{1/4} \left( \frac{\tau^2}{4} \right) & \text{for } \tau > 0 ,
\end{cases} \quad \text{(5)}
\]

with \(L\) the length of the collimator, \(r_p\) the classical particle radius, \(N_b\) the bunch population, \(\gamma\) the relativistic Lorentz factor, \(\lambda = \rho/(120\pi)\), and \(\rho\) the resistivity in \(\mu\Omega\)m. Figure 1 shows the function \(f_R\). Its \(\tau\) average over a Gaussian bunch is \(< f_R >_\tau = 0.816\).

The Piwinski formula (1) applies, if the skin depth \(\delta_s\) fulfills the following conditions [2],

\[
\delta_s \ll \min \left( d_{\text{wall}}, d_{\text{beam-wall}} \frac{1}{\sigma_z^2} \right) , \quad \text{(6)}
\]

where \(d_{\text{wall}}\) denotes the thickness of the wall, \(d_{\text{beam-wall}}\) the distance between the beam center and the wall, and \(k = \omega/c\).

### 2.2 Nonlinear Deflection and Coherent Tune Shift with Finite Beam Size

The nonlinear kick experienced by a test particle at \((x, y)\) due to preceding driving particles at \((x_0, y_0)\) is obtained from the potential by differentiation:

\[
\Delta y' (x, y, x_0, y_0, \tau) = -\frac{\partial V}{\partial y} = -\kappa f_R(\tau) \frac{\partial \Phi}{\partial y} \quad \text{(7)}
\]
Figure 1: The function $f_R(\tau)$, (5), versus $\tau = z/\sigma_z$.

$$= \frac{\pi}{4b^2} \kappa f_R(\tau) \left[ -\pi (y - y_0) \cos \left( \frac{\pi(y-y_0)}{2b} \right) - 2b \sin \left( \frac{\pi(y-y_0)}{2b} \right) \right. $$
$$+ \frac{\pi (y + y_0) \cos \left( \frac{\pi(y+y_0)}{2b} \right) + 2b \sin \left( \frac{\pi(y+y_0)}{2b} \right)}{\cos \left( \frac{\pi(y-y_0)}{2b} \right) + \cosh \left( \frac{\pi(x-x_0)}{2b} \right)} $$
$$- \pi \sin \left( \frac{\pi(y-y_0)}{2b} \right) \left( y - y_0 \sin \left( \frac{\pi(y-y_0)}{2b} \right) + (x - x_0) \sinh \left( \frac{\pi(x-x_0)}{2b} \right) \right) $$
$$\left. \right] $$
$$\approx \kappa f_R(\tau) \left( \frac{\pi^2}{6b^2} \right) (2y_0 + y) , $$

(8)

where the last approximation was made for $x = x_0 = 0$, $y_0 \to 0$, and $y \to 0$. We recognize a factor two difference between the coherent linear wake field (coefficient of $y_0$) and the incoherent linear wake field (coefficient of $y$), consistent with Yokoya’s theory for the resistive-wall wake in a flat chamber [4].

The coherent tune shift including nonlinear and finite-beam size effects can now be calculated by differentiating the deflection $\Delta y'$ with respect to a bunch centroid offset and then integrating over the transverse Gaussian distributions of both test and driving particles (in the vertical direction the distribution is cut off at the gap amplitude):

$$\Delta Q_y = -\frac{1}{F} \frac{\beta}{4\pi} \kappa \langle f_R \rangle \tau \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-b+2y_0}^{b} \int_{-b+2y_0}^{b} G(x, y, x_0, y_0; y_0) $$

4
where

\[
G(x, y, x_0, y_0; y_{co}) = \frac{1}{\kappa F(r)} \int_{y_c = y_{co}} \partial(y'(x, y_c + y, x_0, y_c + y_0, \tau)) \, dy_c \, dx_0 \, dy_0 \, dx, \tag{9}
\]

and

\[
G(x, y, x_0, y_0; y_{co}) = \frac{1}{\kappa F(r)} \int_{y_c = y_{co}} \partial(y'(x, y_c + y, x_0, y_c + y_0, \tau)) \, dy_c \, dx_0 \, dy_0 \, dx.
\tag{10}
\]

with \(y_c\) denoting the offset of the vertical bunch centroid position (taken to be constant along the bunch), i.e., we compute the coherent tune shift from the average restoring force experienced by the bunch in response to a small centroid offset.

In (9), \(F\) is a normalization factor for the bunch charge,

\[
F = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{b+2y_{co}}^{b} \int_{b+2y_{co}}^{b} e^{-\frac{(y-y_{co})^2}{2\sigma_y^2}} \frac{dy \, dy_0 \, dx \, dx_0}{(2\pi)^2 \sigma_x^2 \sigma_y^2},
\]

\[
= \left( \text{erf} \left( \frac{b - y_{co}}{\sqrt{2\sigma_y}} \right) \right)^2,
\tag{11}
\]

and \(y_{co}\) represents a possible offset of the closed orbit from the center of the two collimator jaws, thereby accounting for possible alignment errors. The function \(\text{erf}\) is the error function \(\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt\).

Explicitly, the response function \(G(x, y, x_0, y_0; y_{co})\) of (10) is

\[
G(x, y, x_0, y_0; y_{co}) = -\frac{\pi^2}{8b^3} \left( \cos \left( \frac{\pi Y}{2b} \right) + \cosh \left( \frac{\pi X}{2b} \right) \right)^{-3} \left[ \left\{ 2\pi Y \sin \left( \frac{\pi Y}{2b} \right) - 8b \cos \left( \frac{\pi Y}{2b} \right) \right\} \cosh \left( \frac{\pi X}{2b} \right)^2 

- \left\{ 4b \cos \left( \frac{\pi Y}{b} \right) + 12b + \pi Y \sin \left( \frac{\pi Y}{b} \right) \right\} \cosh \left( \frac{\pi X}{2b} \right) - 4\pi Y \sin \left( \frac{\pi Y}{2b} \right) 

- 8b \cos \left( \frac{\pi Y}{2b} \right) - \pi X \left( \cos \left( \frac{\pi Y}{b} \right) - 3 \right) \sinh \left( \frac{\pi X}{2b} \right) + \pi X \cos \left( \frac{\pi Y}{2b} \right) \sinh \left( \frac{\pi X}{b} \right) \right] ,
\tag{12}
\]

where

\[
X \equiv x - x_0, \quad \text{and} \quad Y \equiv y + y_0 + 2y_{co}.
\tag{13}
\]

We find it convenient to also introduce

\[
U \equiv x + x_0, \quad \text{and} \quad W \equiv y - y_0,
\tag{14}
\]

so that we can express \(x, x_0, y, y_0\) in terms of \(X, Y, U, W\) via

\[
x \equiv \frac{X + U}{2}, \quad x_0 \equiv \frac{U - X}{2},
\]

\[
y \equiv \frac{Y + W}{2} - y_{co}, \quad y_0 \equiv \frac{Y - W}{2} - y_{co}.
\tag{15}
\]

5
After changing the integration variables from \((x, y, x_0, y_0)\) to \((X, Y, U, W)\), (9) becomes

\[
\Delta Q_y = -\frac{1}{F} \frac{\beta}{4\pi} \langle f_R \rangle_+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-2b+2y_0}^{2b} G(X, Y) e^{-\frac{y^2}{4\sigma_y^2} - \frac{z^2}{4\sigma_z^2}} \frac{1}{4} dY \ dW \ dX \ dU ,
\]

(16)

where the reduced function \(G(X, Y)\) depends only on \(X\) and \(Y\):

\[
G(X, Y) \equiv G(x, y, x_0, y_0) \bigg| \begin{array}{c}
  x = x(X, U), x_0 = x_0(X, U) \\
  y = y(Y, W), y_0 = y_0(Y, W)
\end{array} .
\]

(17)

\(G(X, Y)\) is given by the right-hand side of (12). The integrations over \(W\) and \(U\) in (16) are then easily executed, leading to

\[
\Delta Q_{y,\text{non}1}^{\text{flat}} = -\frac{\text{erf} \left( \frac{b-y_0}{\sigma_y} \right)}{F} \frac{\beta}{4\pi} \frac{\langle f_R \rangle_+}{\int_{-2b+4y_0}^{2b} G(X, Y) \ e^{-\frac{y^2}{4\sigma_y^2} - \frac{z^2}{4\sigma_z^2}} \ dY \ dX} .
\]

(18)

We evaluate the remaining two integrals numerically. With the help of the auxiliary function

\[
P(Y) \equiv \int_{-\infty}^{\infty} G(X, Y) \frac{e^{-\frac{y^2}{4\sigma_y^2}}}{\sqrt{2\pi}} dX ,
\]

(19)

and inserting the definition of \(\kappa\), (4), the coherent tune shift (18) can be rewritten as

\[
\Delta Q_{y,\text{non}1}^{\text{flat}} = -\frac{\text{erf} \left( \frac{b-y_0}{\sigma_y} \right)}{\left[ \text{erf} \left( \frac{b-y_0}{\sqrt{2\sigma_y}} \right) \right]^2} \frac{\beta}{16\pi\gamma^2} \frac{N r_p L \sqrt{\lambda}}{\sigma_z b} \int_{-2b+4y_0}^{2b} P(Y) \ e^{-\frac{y^2}{4\sigma_y^2}} dY .
\]

(20)

2.3 Example

As an illustration, we consider the example parameter set listed in Table 1, which represents a typical experimental condition at the SPS. Figure 2 shows the behavior of the function \(G(X, Y)\), as a function of the two arguments \(X\) and \(Y\). Note that the function \(G\) changes sign for large arguments \(X\) (particles in these regions suffer a tune shift with a sign opposite to that of particles near the center), and that it diverges for \(Y \to 2b\). The function \(P(Y)\) of (19) is depicted in Fig. 3 for the nominal value of \(\sigma_z\) and for a ten times smaller one. Figure 4 displays the coherent tune shift \(\Delta Q_{y,\text{non}1}^{\text{flat}}\), (20), as a function of the transverse emittance. In the limit of zero emittance, it approaches the classical value \(\Delta Q_{\text{flat}}^{\text{class}} \approx -2.30 \times 10^{-4}\), calculated from Eq. (37). For comparison, the values computed from the round-beam Burov-Lebedev formula [5] with Yokoya factor [4], (28), is \(\Delta Q_{\text{BL},1}^{\text{flat}} \approx -2.17 \times 10^{-4}\), and from the approximate flat-chamber Burov-Lebedev formula [6], (31), \(\Delta Q_{\text{BL},2}^{\text{flat}} \approx -2.23 \times 10^{-4}\).
Table 1: Example parameters; shown in parentheses are the parameters of the actual SPS experiment where they differ from the example values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>bunch population</td>
<td>$N_b$</td>
<td>$10^{11}$</td>
</tr>
<tr>
<td>horizontal beta function</td>
<td>$\beta_x$</td>
<td>93.0 m</td>
</tr>
<tr>
<td>vertical beta function</td>
<td>$\beta_y$</td>
<td>25 (23.8) m</td>
</tr>
<tr>
<td>dispersion function</td>
<td>$D_y$</td>
<td>0 (22) cm</td>
</tr>
<tr>
<td>normalized horizontal emittance</td>
<td>$\gamma\epsilon_x$</td>
<td>1.5 $\mu$m</td>
</tr>
<tr>
<td>normalized vertical emittance</td>
<td>$\gamma\epsilon_y$</td>
<td>1.5 $\mu$m</td>
</tr>
<tr>
<td>rms horizontal beam size</td>
<td>$\sigma_x$</td>
<td>0.72 mm</td>
</tr>
<tr>
<td>rms vertical beam size</td>
<td>$\sigma_y$</td>
<td>0.37 (0.39) mm</td>
</tr>
<tr>
<td>rms bunch length</td>
<td>$\sigma_z$</td>
<td>0.21 m</td>
</tr>
<tr>
<td>rms energy spread</td>
<td>$\sigma_\delta$</td>
<td>$0.6 \times 10^{-3}$</td>
</tr>
<tr>
<td>rf voltage</td>
<td>$V_{rf}$</td>
<td>7 MV</td>
</tr>
<tr>
<td>momentum compaction</td>
<td>$\alpha_c$</td>
<td>$1.856 \times 10^{-3}$</td>
</tr>
<tr>
<td>circumference</td>
<td>$C$</td>
<td>6916 m</td>
</tr>
<tr>
<td>vertical tune</td>
<td>$Q_y$</td>
<td>26.135</td>
</tr>
<tr>
<td>proton beam momentum</td>
<td>$p$</td>
<td>270 GeV/c</td>
</tr>
<tr>
<td>collimator half gap</td>
<td>$b$</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>collimator thickness</td>
<td>$d$</td>
<td>30 mm</td>
</tr>
<tr>
<td>collimator resistivity</td>
<td>$\rho$</td>
<td>$10 \mu\Omega$ m</td>
</tr>
</tbody>
</table>
Figure 2: The function $G(X, Y)$ of (17) for $b = 1.5$ mm: contour plot in $X - Y$ plane (top left); plot as a function of $X$ for various values of $Y$ (top right); and plot as a function of $Y$ for various values of $X$ (bottom).
2.4 Incoherent Tune Spread

The formula for the incoherent tune shift is similar to (9), namely

$$\Delta Q_{y,\text{inc}}(x, y, \tau) = \frac{1}{F} \frac{\beta}{4\pi} \kappa f_R(\tau) \int_{-\infty}^{\infty} \int_{b}^{b} \tilde{G}(x, y, x_0, y_0) e^{\frac{y_0^2}{2\sigma_y^2} - \frac{x_0^2}{2\sigma_x^2}} \frac{dx_0}{(2\pi)\sigma_x \sigma_y} dy_0 \, dx_0 ,$$  

(21)

where

$$\tilde{G}(x, y, x_0, y_0) = \kappa f_R(\tau) \frac{\partial(\Delta y'(x, y, x_0, y_0, \tau))}{\partial y}$$  

(22)

The derivative of the nonlinear kick is now taken only with respect to the vertical coordinate of the test particle. In addition, the dependence on the single-particle coordinate $\tau$, given by $f_R(\tau)$, contributes to the incoherent tune spread. Further, in writing (21), we have ignored the possibility of a closed-orbit offset. Explicitly, the function $G$ of (22) is

$$\tilde{G}(x, y, x_0, y_0)$$
Figure 4: Coherent tune shift $\Delta Q_{y,\text{nonlin}}^{\text{flat}}$ computed from the nonlinear formulae (18) or equivalently (20), as a function of the two transverse normalized emittances, for $\beta_x = 93$ m, $\beta_y = 25$ m, $\gamma = 27.7$, a vertical half gap of 1.5 mm, and without closed orbit offset ($y_{\text{co}} = 0$). The linear estimate based on the flat-wall Burov-Lebedev formula, (31), and the one from the classical theory, (37), are also indicated. As can be seen, the latter two theories are strictly applicable only in the limit of vanishing emittance, where the nonlinear contributions disappear.

\begin{equation}
\Delta Q_{y,\text{nonlin}}^{\text{flat}} = \frac{\pi^2}{4\beta^2} \left[ \frac{2 \sin y_- (y_- \cos y_- + \sin y_-)}{(\cos y_- - \cosh x_-)^2} + \frac{2 \cos y_- + y_- \sin y_-}{(\cos y_- - \cosh x_-)^2} \right. \\
+ \frac{2 \cos y_+ y_+ \cosh x_-}{(\cos y_- + \cosh x_-)^2} + \frac{2 \cos y_+ - y_+ \sin y_+}{(\cos y_- + \cosh x_-)^2} \\
- \frac{\cos y_- (y_- \sin y_- + x_- \sinh x_-)}{(\cos y_- - \cosh x_-)^2} - \frac{2 \sin^2 y_- (y_- \sin y_- + x_- \sinh x_-)}{(\cos y_- - \cosh x_-)^2} \\
+ \frac{\cos y_+ (y_+ \sin y_+ - x_- \sinh x_-)}{(\cos y_- + \cosh x_-)^2} + \frac{2 \sin^2 y_+ (y_+ \sin y_+ - x_- \sinh x_-)}{(\cos y_- + \cosh x_-)^2} \right], \tag{23}
\end{equation}

with $y_+, y_-$ and $x_-$ as defined in (3).

The incoherent tune distribution (21) can be evaluated by a Monte-Carlo technique, where we randomly choose the coordinates $x$, $y$ and $\tau$ according to the respective Gaussian distribution. Figure 5 displays some incoherent tune distributions so obtained, for two different collimator gaps. The parameters of Table 1 were assumed for this calculation, except for the gap size, which is being varied.
In parallel particle tracking of a three dimensional distribution with the wake potential (1) has also been performed. Centroid motion and its frequency spectrum from a harmonic analysis are shown in Figs. 6 and 7, respectively. The various pictures refer to cases without and with collimator, assuming two different gaps, to cases with and without synchrotron motion, and two different numbers of tracked particles. The result strongly depends on the collimator gap, on the presence of synchrotron motion, and on the number of particles in the tracking.

The increased tune spread for small gap sizes, visible both in Fig. 7, is compatible with the Monte-Carlo estimate of our analytical expression in Fig. 5. The additional tune spread introduced by the collimator may explain the reduction of the coherent signal which was observed by F. Caspers and T. Kroyer with a 245-MHz pick up [7].
Figure 6: Centroid of a three-dimensional particle distribution with an initial offset of 1 μm as a function of turn number, without a collimator (top left), and with a collimator of half gap 1.5 mm (top center) or 1 mm (top right), without synchrotron motion. Results with synchrotron motion are also shown for a gap of 1.5 mm (bottom left) and 1 mm (bottom center). In all these figures, 2000 particles were tracked over 11,000 turns. The last picture shows a result for a gap of 1 mm, where only 1000 particles are tracked without synchrotron motion (bottom right). Comparison with the top right picture illustrates the sensitivity to the particle number.
Figure 7: Tune spectrum of the centroid motion for a three-dimensional particle distribution with an initial offset of 1 μm as a function of turn number, without a collimator (top left), and with a collimator of half gap 1.5 mm (top center) or 1 mm (top right), without synchrotron motion. Results with synchrotron motion are also shown for a gap of 1.5 mm (bottom left) and 1 mm (bottom center). In all these figures, 2000 particles were tracked over 11,000 turns. The last picture shows a result for a gap of 1 mm, where only 1000 particles are tracked without synchrotron motion (bottom right). Comparison with the top right picture illustrates the sensitivity to the particle number.
3 Comparison of Predictions and Experiment

The measured values of bunch length, emittance, and intensity as well as the known collimator half gap sizes are listed in Appendix C. Errors for the emittance, bunch length, and intensity are estimated from the statistical variation of the measurements. In addition, there exists at 10% systematic uncertainty for the absolute current value. Very likely the actual bunch charge is underestimated. More details on the experimental data can be found in Appendix C. The beta functions at the collimator, the beam momentum, the collimator resistivity, and the collimator thickness for the SPS experiment are the same as those given in Table 1. For the vertical beta function and dispersion we now assume the values listed in parentheses. (We here recall that, more precisely, in the actual experiment, the collimator was a horizontal one, the horizontal tune shift was measured, and all horizontal and vertical beam sizes, beta functions and dispersion functions were interchanged, with respect to the table.)

From these various input parameters we compute the tune shift predicted by the nonlinear theory, according to (20). An “error of the theoretical prediction” is obtained by re-computing the tune shifts with one of four parameters — horizontal emittance, vertical emittance, bunch length, or intensity — changed from its nominal (average) value to the nominal value plus the error. In the case of the intensity there are two errors, the second one representing the systematic uncertainty. We then obtain 4 statistical errors, which we add in quadrature, and one systematic error. The results for 10 individual data sets are summarized in Table 2 which also includes the measured values and the predictions from two linear theories — Burov-Lebedev and classical (see Appendices A and B). For the latter two, we have estimated errors in the same way as for the nonlinear theory, except for that the emittance uncertainty does not enter. The relative importance of the various contributions to the overall statistical error can be judged from Table 3.

Figure 8 compares the tune shift predicted by the nonlinear wake-field theory, (20), with measured data from the SPS experiment. Both are plotted as a function of the full gap size $2b$. The maximum deviation is of order 20% for small gap sizes. To quantify the quality of the agreement we define a $\chi^2$ function as

$$\chi^2 = \frac{1}{N_{\text{data}}} \sum_{N_{\text{data}}} \frac{(\Delta Q_{\text{theory}} - \Delta Q_{\text{meas}})^2}{\sigma^2_{\text{theory,stat}} + \sigma^2_{\text{meas}}},$$

where $\sigma_{\text{theory,stat}}$ denotes the error of the theoretical prediction arising from the statistical variation in the measured bunch lengths, emittances and bunch intensity, and $\sigma_{\text{meas}}$ represents the statistical or resolution error of the tune-shift measurement. In this definition we do not take into account the suspected systematic uncertainty in the beam-intensity measurement.

In Fig. 8, the error bar for the predicted values represents both the statistical and the systematic part, in order to reveal the impact of the large systematic uncertainty. The fit quality, computed with the statistical error only, is characterized by $\chi^2 = 0.80$. The measured values lie within “1σ” from the expectation.

Figure 9, which otherwise refers to the same case as Fig. 8, assumes a 50 $\mu$m beam offset from the center of the chamber. The data points look similar, but the fit quality is slightly degraded from $\chi^2 = 0.80$ in Fig. 8 to $\chi^2 = 0.83$ in Fig. 9.
Table 2: Measured tune shift for various collimator gaps compared with predictions from the nonlinear, Burov-Lebedev, and classical theory. Statistical and systematic errors are also indicated.

<table>
<thead>
<tr>
<th>half gap [mm]</th>
<th>measurement $\Delta Q_{\text{meas}} [10^{-4}]$</th>
<th>nonlinear theory $-\Delta Q_{\text{nonlinear}} [10^{-4}]$</th>
<th>Burov-Lebedev $-\Delta Q_{\text{BL}}^{\text{flat,2}} [10^{-4}]$</th>
<th>classical $-\Delta Q_{\text{class}} [10^{-4}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.43 ± 0.05</td>
<td>0.48 ± 0.11(stat.)</td>
<td>$+0.04$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.05$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.05$ (syst.)</td>
</tr>
<tr>
<td>1.93 ± 0.05</td>
<td>0.76 ± 0.11(stat.)</td>
<td>$+0.08$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.09$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.10$ (syst.)</td>
</tr>
<tr>
<td>1.43 ± 0.05</td>
<td>1.37 ± 0.15(stat.)</td>
<td>$+0.17$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.21$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.24$ (syst.)</td>
</tr>
<tr>
<td>1.13 ± 0.05</td>
<td>2.14 ± 0.13(stat.)</td>
<td>$+0.25$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.34$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.40$ (syst.)</td>
</tr>
<tr>
<td>0.98 ± 0.05</td>
<td>2.42 ± 0.12(stat.)</td>
<td>$+0.29$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.43$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.51$ (syst.)</td>
</tr>
<tr>
<td>1.93 ± 0.05</td>
<td>0.63 ± 0.18(stat.)</td>
<td>$+0.08$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.09$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.10$ (syst.)</td>
</tr>
<tr>
<td>1.43 ± 0.05</td>
<td>1.47 ± 0.13(stat.)</td>
<td>$+0.18$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.21$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.23$ (syst.)</td>
</tr>
<tr>
<td>1.23 ± 0.05</td>
<td>1.95 ± 0.09(stat.)</td>
<td>$+0.25$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.30$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.35$ (syst.)</td>
</tr>
<tr>
<td>1.03 ± 0.05</td>
<td>2.72 ± 0.12(stat.)</td>
<td>$+0.34$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.45$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.53$ (syst.)</td>
</tr>
<tr>
<td>0.93 ± 0.05</td>
<td>2.64 ± 0.12(stat.)</td>
<td>$+0.37$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.52$ (syst.) $-0.00$ (syst.)</td>
<td>$+0.62$ (syst.)</td>
</tr>
</tbody>
</table>
Table 3: Contributions to the statistical error for the nonlinear theory.

<table>
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<tr>
<th>half gap [mm]</th>
<th>$\Delta (\Delta Q)^{\epsilon z}[10^{-5}]$</th>
<th>$\Delta (\Delta Q)^{\epsilon y}[10^{-5}]$</th>
<th>$\Delta (\Delta Q)^{\epsilon z}[10^{-5}]$</th>
<th>$\Delta (\Delta Q)^{N_h}[10^{-5}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.43</td>
<td>0.01</td>
<td>0.05</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>1.93</td>
<td>0.03</td>
<td>0.12</td>
<td>0.11</td>
<td>0.02</td>
</tr>
<tr>
<td>1.43</td>
<td>0.10</td>
<td>0.37</td>
<td>0.23</td>
<td>0.17</td>
</tr>
<tr>
<td>1.13</td>
<td>0.76</td>
<td>0.67</td>
<td>0.34</td>
<td>1.54</td>
</tr>
<tr>
<td>0.98</td>
<td>0.77</td>
<td>0.87</td>
<td>0.40</td>
<td>1.42</td>
</tr>
<tr>
<td>1.93</td>
<td>0.04</td>
<td>0.15</td>
<td>0.09</td>
<td>0.02</td>
</tr>
<tr>
<td>1.43</td>
<td>0.13</td>
<td>0.47</td>
<td>0.19</td>
<td>0.11</td>
</tr>
<tr>
<td>1.23</td>
<td>0.21</td>
<td>0.75</td>
<td>0.26</td>
<td>0.73</td>
</tr>
<tr>
<td>1.03</td>
<td>1.22</td>
<td>1.17</td>
<td>0.35</td>
<td>0.59</td>
</tr>
<tr>
<td>0.93</td>
<td>1.03</td>
<td>1.34</td>
<td>0.38</td>
<td>1.65</td>
</tr>
</tbody>
</table>

The measurement data can also be compared with predictions from two other theories, which do not include the nonlinear component of the wake field. Figure 10 shows the prediction by the Burov-Lebedev theory for a flat chamber, described in Appendix A. The Burov-Lebedev formalism takes into account the finite thickness of the collimator jaw. For small gap sizes, we observe larger discrepancies, and with $\chi^2 = 5.15$ the quality of agreement with the measurement is more than 5 times worse than for the nonlinear theory in Fig. 8.

Figure 11 shows the prediction from classical theory (Appendix B), which ignores both the chamber thickness and the nonlinear wake field components. The disagreement with the measurement is even larger than in Fig. 10, and the quality factor increases to $\chi^2 = 8.73$.

The nonlinear wake field formula for an infinitely thick wall, depicted in Fig. 8, yields the predictions closest to the experimental data points. The remaining difference from the measurement is of the order 20% for the smallest gap sizes and less for larger gaps. On the other hand, the difference between the Burov-Lebedev formula and the classical theory, shown as predictions in Figs. 10 and 11, respectively, is also about 20%. Therefore, it appears likely that a further generalized theory which would include both the nonlinearity of the wake fields and the finite chamber thickness might yield a perfect agreement with the measurement. This question is explored in the next section.
Figure 8: Measured data points (red rhombi) compared with prediction from the nonlinear-wake theory (20) (blue triangles). The error bar on the predicted values contains both the statistical and the systematic error. The quality of the agreement, considering the statistical error only, is characterized by $\chi^2 = 0.80$.

Figure 9: Measured data points (red rhombi) compared with prediction from the nonlinear-wake theory (20) (blue triangles), assuming that the closed orbit is offset by $50 \mu m$ from the center of the collimator. The error bar on the predicted values contains both the statistical and the systematic error. The quality of the agreement, considering the statistical error only, is characterized by $\chi^2 = 0.83$. 

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Figure 10: Measured data points (red rhombi) compared with prediction from the flat-chamber Burov-Lebedev formula (31) (blue triangles). The error bar on the predicted values contains both the statistical and the systematic error. The quality of the agreement, considering the statistical error only, is characterized by $\chi^2 = 5.15$.

Figure 11: Measured data points (red rhombi) compared with prediction from the classical formula (37) (blue triangles). The error bar on the predicted values contains both the statistical and the systematic error. The quality of the agreement, considering the statistical error only, is characterized by $\chi^2 = 8.73$. 

18
Towards a General Theory

The Burov-Lebedev theory describes the correct time dependence of the wake field, including so-called inductive bypass phenomena at low frequencies, while the Piwinski theory represents the complete nonlinear dependence of the wake field on the transverse coordinates of driving and probe particles. Assuming that the two dependencies remain factorized, we can combine the two theories. Namely, we augment the Burov-Lebedev formula for a flat-chamber 31 by a multiplicative correction term for the nonlinear transverse components, whose coefficient we infer from the linear expansion in (8). This gives

\[
\Delta Q^\text{general}_y \approx \frac{\beta}{4\pi} \frac{N_b \tau p L}{2\pi \gamma} \int_{-\infty}^{\infty} \frac{3}{2} Z_{\text{BL},y}^{\text{Flat},2} (\omega) \, e^{-\omega^2 \sigma_r^2 / c^2} \, d\omega \left( \frac{3b^2}{\pi^2} \right) \, \text{erf} \left( \frac{b-b_0}{\sigma_y} \right) \int_{-\infty}^{\infty} \int_{-2b+4\gamma_o}^{2b} G(X, Y) \, e^{-\frac{X^2}{4\sigma_x^2}-\frac{Y^2}{4\sigma_y^2}} \, dY \, dX, \tag{25}
\]

where the function \( G(X, Y) \) is given by the right-hand side of (12).

In Fig. 12, the prediction of (25) is compared with the measurement. With a fit quality \( \chi^2 \) of 0.15, from all theories this combined one yields the closest agreement with the experiment.

**Figure 12:** Measured data points (red rhombi) compared with prediction from the combined Burov-Lebedev and nonlinear-wake theory (25) (blue triangles). The error bar on the predicted values contains both the statistical and the systematic error. The quality of the agreement, considering the statistical error only, is characterized by \( \chi^2 = 0.15 \).
5 Conclusions

We have developed analytical expressions for the coherent tune shift and the incoherent tune spread induced by a collimator whose gap is of similar magnitude as the transverse beam size, taking into account the nonlinear components of the resistive-wall wake field in a flat chamber, through infinite order. Our formalism is based on the nonlinear wake potential derived by Piwinski [2].

The tune shifts predicted for the conditions of the 2004 LHC collimator experiment in the SPS are 20% higher than the measured values, for small collimator gaps. The prediction agrees with the measurement within the statistical errors. In particular, the predicted tune shift values are 30–50% lower — and, thereby, lie much closer to the experimental points — than those from conventional theories [5, 6, 8], which include only the linear part of the wake field. The remaining discrepancy of about 20% appears to be due to the effect of the finite jaw thickness. Indeed, by combining the Piwinski theory for the nonlinear dependence on the transverse coordinates of drive and test particles with the time dependence of the Burov-Lebedev theory, we constructed a general formula, (25), whose predictions are nearly indistinguishable from the measured data points, for all values of the collimator gap (Fig. 12).

At the smallest collimator gaps the incoherent tune spread increases significantly, which might explain a reduction of the coherent beam-motion signal observed for small gaps [7].

Acknowledgements

It is a pleasure to thank Gianluigi Arduini, Ralph Assmann, Helmut Burkhardt, Fritz Caspers, Marek Gasior, Thomas Kroyer, Elias Métral, Stefano Redaelli, and Federico Roncarolo for providing the experimental data, or collaborating in their collection, and for helpful discussions. I thank G. Arduini and E. Métral also for a careful reading of the manuscript.
References


A Burov-Lebedev Theory of Linear Resistive-Wall Wake Field

The theory developed by Burov and Lebedev [5, 6] includes the effect of the finite chamber thickness, but considers only the linear wake field. It assumes $c/\omega >> b$ and also $\beta\gamma >> 1$ (relativistic limit; for an estimate of non-relativistic corrections see [9]). In the Burov-Lebedev theory for a round chamber of thickness $d$ with inner radius $b$ and outer radius $a = (b + d)$, surrounded by vacuum, the impedance is [5]

$$Z_{\text{BL},1}^{\text{round}} = -\frac{Z_0}{\pi b^2} \frac{1 + \tilde{\kappa}_{21} s_1 / s_1'}{1 + \tilde{\kappa}_{21} c_1 / s_1' + \tilde{\kappa}_{21} s_1 / s_1' + \tilde{\kappa}_{10} c_1 / s_1'}, \quad (26)$$

where

$$\tilde{\kappa}_{21} = \tilde{\kappa}_2 / \tilde{\kappa}_1, \quad \tilde{\kappa}_{10} = \tilde{\kappa}_1 b,$$

$$\tilde{\kappa}_1 = \kappa / \mu, \quad \tilde{\kappa}_2 = 1 / a,$$

$$\kappa = \sqrt{-4\pi i \sigma \mu \omega / c^2},$$

$$c_1 = \text{chb}(\kappa a), \quad s_1 = \text{shb}(\kappa a),$$

$$c_1' = \text{chb}'(\kappa a), \quad s_1' = \text{shb}'(\kappa a),$$

$$\text{shb}(\kappa r) = \kappa b [I_1(\kappa r) K_1(\kappa b) - I_1(\kappa b) K_1(\kappa r)], \quad \text{chb}(\kappa r) = \kappa b [I_1'(\kappa r) K_1(\kappa b) - I_1(\kappa b) K_1'(\kappa r)].$$
Except for extremely low frequencies, where \( \delta_s > b \) (with \( \delta_s \) the skin depth), the impedance (26) can further be approximated by [5]

\[
Z_{\text{round},2}^{\text{BL}} = -i \frac{Z_0}{\pi b^2} \frac{1 + \kappa_{21}t_1}{1 + \kappa_{21} \kappa_{12} + (\kappa_{21} + \kappa_{10})t_1},
\]

(27)

with \( t_1 = \tanh(\kappa_{1}d) \). The impedance of a flat chamber is often approximated by multiplying the impedance of the round chamber with the Yokoya factor \( \pi^2/8 \) [4], giving the tune shift

\[
\Delta Q_{\text{BL,y}}^{\text{flat,1}} \approx \frac{\beta}{\pi} \frac{\pi^2}{8} \frac{N_{b^f}}{2\pi \gamma} \int_{-\infty}^{\infty} Z_{\text{BL}}^{\text{round,2}}(\omega) e^{-\omega^2 \sigma_y^2 / \gamma} d\omega.
\]

(28)

However, the exact impedance solution for the flat chamber is slightly different, namely [6]

\[
Z_{\text{BL,y}}^{\text{flat,2}} \approx -i \frac{Z_0}{2\pi b^2} \int_0^\infty d\xi \frac{\xi^2}{\sinh \xi (\xi e^\xi + \tau \sinh \xi)} d\xi,
\]

(29)

where \( \tau = \kappa b \tanh(\kappa d) \), and \(|\kappa|b >> 1\) is assumed. With an accuracy better than 13\%, (29) can further be approximated as

\[
Z_{\text{BL,y}}^{\text{flat,2}} \approx -i \frac{\pi^2}{12} \frac{Z_0}{2\pi b^2} \frac{1}{1 + \tau/2},
\]

(30)

and, taking into account the incoherent contribution to the coherent tune shift for a flat chamber [10], the corresponding tune shift follows from

\[
\Delta Q_{\text{BL,y}}^{\text{flat,2}} \approx \frac{\beta}{4\pi} \frac{N_{b^f} L}{2\pi \gamma} \int_{-\infty}^{\infty} \frac{3}{2} \frac{Z_{\text{BL,y}}^{\text{flat,2}}(\omega)}{Z_{\text{BL}}^{\text{round,2}}(\omega)} e^{-\omega^2 \sigma_y^2 / \gamma} d\omega.
\]

(31)
The classical theory of the resistive wall impedance [8] is valid for a wall much thicker than the skin depth
\[ d \gg \delta \, ; \] (32)
and for
\[ \frac{\chi c}{b} \ll |\omega| \ll \frac{c}{b\chi^{1/3}} \, , \] (33)
with
\[ \chi = \frac{1}{Z_0\sigma b} \, . \] (34)

\( Z_0 \approx 377 \Omega \) the vacuum impedance, and \( \sigma \) the conductivity of the wall. Typically, \( \chi \) is small. For example, we find \( \chi \approx 2 \times 10^{-5} \) for carbon of \( \sigma \approx 10^5 \, \Omega^{-1} \, \text{m}^{-1} \) with half gap \( b \approx 1.5 \, \text{mm} \), and the approximation is good over a large frequency range, from about 1 MHz to 1 THz.

Introducing
\[ \lambda_0(\omega) = (i + \text{sgn}(\omega))\sqrt{\frac{\mu_0\sigma |\omega|}{2}} \, , \] (35)
the impedance is
\[ Z_{\text{class}}(\omega) = -\frac{i}{b^3} \frac{4\pi \lambda_0(\omega) c}{Z_0\sigma c \, \omega} \, , \] (36)
and, for a flat chamber with half height \( b \), the classical coherent tune shift becomes
\[ \Delta Q_{\text{class}}^{\text{flat}} \approx \frac{\beta}{4\pi} \frac{\pi}{8} \frac{N_b r_p L}{\gamma} \int_{-\infty}^{\infty} Z_{\text{class}}(\omega) \, e^{-\omega^2 \sigma_z^2/c^2} \, d\omega \, . \] (37)

The factor \((\pi^2/8)\) describes the difference between a flat and a round chamber, calculated by K. Yokoya [4]. As for the Burov-Lebedev theory, the classical expression is independent of the beam emittance.
C Experimental Data

An experiment with a prototype LHC collimator was performed at a proton beam momentum of 270 GeV/c in the CERN SPS on 12.10.2004. The tune shifts were measured as a function of the collimator gap size by Marek Gasior, with results as listed in Table 4, and illustrated in Fig. 13. Error bars in the horizontal and vertical direction are indicated.

Table 4: Tune shifts measured during collimator machine studies on 12.10.2004 with errors, from Marek Gasior.

<table>
<thead>
<tr>
<th>time (readme file)</th>
<th>time (Marek)</th>
<th>sc number</th>
<th>gap [mm] (readme)</th>
<th>gap [mm] (Marek)</th>
<th>error [mm]</th>
<th>ΔQ [Hz]</th>
<th>error [Hz]</th>
<th>ΔQ [10⁻³]</th>
<th>error [10⁻³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>03:30</td>
<td>03:35:42</td>
<td>30650–30671</td>
<td>4.8</td>
<td>4.86</td>
<td>0.05</td>
<td>2.07</td>
<td>0.47</td>
<td>0.0477</td>
<td>0.0108</td>
</tr>
<tr>
<td>03:41</td>
<td>03:45:28</td>
<td>30672–30690</td>
<td>3.8</td>
<td>3.86</td>
<td>0.05</td>
<td>3.28</td>
<td>0.49</td>
<td>0.0756</td>
<td>0.0113</td>
</tr>
<tr>
<td>03:50</td>
<td>04:01:57</td>
<td>30690–30694</td>
<td>2.8</td>
<td>2.86</td>
<td>0.05</td>
<td>5.96</td>
<td>0.64</td>
<td>0.1373</td>
<td>0.0147</td>
</tr>
<tr>
<td>–03:52</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>04:14</td>
<td>04:21:49</td>
<td>30738–30778</td>
<td>2.1</td>
<td>2.26</td>
<td>0.05</td>
<td>9.28</td>
<td>0.57</td>
<td>0.2138</td>
<td>0.0131</td>
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<td>(10% losses)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>04:34</td>
<td>04:39:44</td>
<td>30779–30794</td>
<td>1.9</td>
<td>1.96</td>
<td>0.05</td>
<td>10.49</td>
<td>0.50</td>
<td>0.2417</td>
<td>0.0115</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>05:25</td>
<td>05:29:56</td>
<td>30879–30897</td>
<td>3.8</td>
<td>3.86</td>
<td>0.05</td>
<td>2.73</td>
<td>0.77</td>
<td>0.0629</td>
<td>0.0177</td>
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<tr>
<td>05:34</td>
<td>05:37:34</td>
<td>30898–30913</td>
<td>2.8</td>
<td>2.86</td>
<td>0.05</td>
<td>6.38</td>
<td>0.549</td>
<td>0.1470</td>
<td>0.0126</td>
</tr>
<tr>
<td>05:42</td>
<td>05:45:51</td>
<td>30914–30929</td>
<td>2.4</td>
<td>2.46</td>
<td>0.05</td>
<td>8.48</td>
<td>0.37</td>
<td>0.1954</td>
<td>0.0085</td>
</tr>
<tr>
<td>(2% losses)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>05:50</td>
<td>05:53:19</td>
<td>30930–30943</td>
<td>2.0</td>
<td>2.06</td>
<td>0.05</td>
<td>11.82</td>
<td>0.53</td>
<td>0.2723</td>
<td>0.0122</td>
</tr>
<tr>
<td>(6% losses)</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>05:57</td>
<td>06:04:37</td>
<td>30944–30973</td>
<td>1.8</td>
<td>1.86</td>
<td>0.05</td>
<td>11.46</td>
<td>0.54</td>
<td>0.2640</td>
<td>0.0124</td>
</tr>
<tr>
<td>–06:12</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The emittances measured by wire scans increased during the store time, partly as a result of these very wire scans. The blow up is evident from the emittance data in Table 5, which were provided by Federico Roncarolo. For the smallest gaps some scraping occurred, and the intensity dropped. The intensity data, recovered with the help of Gianluigi Arduini, are summarized in Table 6. The bunch length was measured with a wide-band pick up, with results shown in Fig. 14. Error bars are listed for emittances, intensities, and bunch length in Tables 5 and 6, and in the caption of Fig. 14, respectively. For the intensity, measured by a fast beam current transformer, the
statistical error refers to the variation of the measured values over the data taken at one particular
gap size and store period. A large systematic error indicates the absolute uncertainty, which was
estimated by comparing the fast and dc current transformer readings [11]. The reading of the fast
current transformer is about 10% lower than the dc current measurement, both at injection and
after acceleration.
Table 5: Emittances measured during collimator machine studies on 12.10.2004 with errors, from Federico Roncarolo.

<table>
<thead>
<tr>
<th>Time</th>
<th>Time (Marek)</th>
<th>Sc number</th>
<th>$\gamma \epsilon_x$ ($\mu$m)</th>
<th>$\Delta (\gamma \epsilon_x)$ error</th>
<th>$\gamma \epsilon_y$ ($\mu$m)</th>
<th>$\Delta (\gamma \epsilon_y)$ error</th>
</tr>
</thead>
<tbody>
<tr>
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<td>03:35:42</td>
<td>30650–30671</td>
<td>1.0</td>
<td>0.1</td>
<td>1.3</td>
<td>0.1</td>
</tr>
<tr>
<td>03:41</td>
<td>03:45:28</td>
<td>30672–30690</td>
<td>1.25</td>
<td>0.1</td>
<td>1.4</td>
<td>0.1</td>
</tr>
<tr>
<td>03:50–03:52</td>
<td>04:01:57</td>
<td>30690–30694</td>
<td>1.5</td>
<td>0.1</td>
<td>1.5</td>
<td>0.1</td>
</tr>
<tr>
<td>04:14</td>
<td>04:21:49</td>
<td>30738–30778</td>
<td>2.0</td>
<td>0.3</td>
<td>1.7</td>
<td>0.1</td>
</tr>
<tr>
<td>04:34–04:42</td>
<td>04:39:44</td>
<td>30779–30794</td>
<td>2.3</td>
<td>0.2</td>
<td>1.8</td>
<td>0.1</td>
</tr>
<tr>
<td>05:25</td>
<td>05:29:56</td>
<td>30879–30897</td>
<td>1.0</td>
<td>0.1</td>
<td>0.9</td>
<td>0.1</td>
</tr>
<tr>
<td>05:34</td>
<td>05:37:34</td>
<td>30898–30913</td>
<td>1.1</td>
<td>0.1</td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>05:42</td>
<td>05:45:51</td>
<td>30914–30929</td>
<td>1.2</td>
<td>0.1</td>
<td>1.1</td>
<td>0.1</td>
</tr>
<tr>
<td>05:50</td>
<td>05:53:19</td>
<td>30930–30943</td>
<td>1.3</td>
<td>0.3</td>
<td>1.2</td>
<td>0.1</td>
</tr>
<tr>
<td>05:57–06:12</td>
<td>06:04:37</td>
<td>30944–30973</td>
<td>1.3</td>
<td>0.2</td>
<td>1.25</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 6: Intensities measured during collimator machine studies on 12.10.2004 with errors, from Gianluigi Arduini.

<table>
<thead>
<tr>
<th>Time</th>
<th>Time (Marek)</th>
<th>Sc number</th>
<th>Intensity ($10^9$)</th>
<th>Intensity error-stat. [$10^9$]</th>
<th>Intensity error-syst. [$10^9$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>03:30</td>
<td>03:35:42</td>
<td>30650–30671</td>
<td>95.72</td>
<td>0.20</td>
<td>+9.6/−0</td>
</tr>
<tr>
<td>03:41</td>
<td>03:45:28</td>
<td>30672–30690</td>
<td>94.91</td>
<td>0.25</td>
<td>+9.5/−0</td>
</tr>
<tr>
<td>03:50–03:52</td>
<td>04:01:57</td>
<td>30690–30694</td>
<td>93.58</td>
<td>0.95</td>
<td>+9.4/−0</td>
</tr>
<tr>
<td>04:14</td>
<td>04:21:49</td>
<td>30738–30778</td>
<td>77.15</td>
<td>4.77</td>
<td>+7.7/−0</td>
</tr>
<tr>
<td>04:34–04:42</td>
<td>04:39:44</td>
<td>30779–30794</td>
<td>64.37</td>
<td>3.08</td>
<td>+6.4/−0</td>
</tr>
<tr>
<td>05:25</td>
<td>05:29:56</td>
<td>30879–30897</td>
<td>91.47</td>
<td>0.17</td>
<td>+9.1/−0</td>
</tr>
<tr>
<td>05:34</td>
<td>05:37:34</td>
<td>30898–30913</td>
<td>89.63</td>
<td>0.55</td>
<td>+9.0/−0</td>
</tr>
<tr>
<td>05:42</td>
<td>05:45:51</td>
<td>30914–30929</td>
<td>85.54</td>
<td>0.247</td>
<td>+8.6/−0</td>
</tr>
<tr>
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<td>05:53:19</td>
<td>30930–30943</td>
<td>77.10</td>
<td>1.32</td>
<td>+7.7/−0</td>
</tr>
<tr>
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<td>06:04:37</td>
<td>30944–30973</td>
<td>66.16</td>
<td>2.94</td>
<td>+6.6/−0</td>
</tr>
</tbody>
</table>
Figure 14: Fitted rms bunch lengths during coasts 6 and 7. The average bunch lengths are $0.691 \pm 0.019$ ns and $0.675 \pm 0.014$ ns, respectively. In coast 6, from 3.5 h to 3.683 h, the full gap was 4.86 mm, from 3.683 h to 3.833 h it was 3.86 mm, from 3.833 h to 0.387 h it was 2.86 mm, from 4.233 h to 4.567 h it was 2.26 mm, and from 4.567 h to 4.7 h it was 1.96 mm. The rf tripped at 3.867 h. In coast 7, from 5.83 h to 5.95 h, the full gap was 2.06 mm, from 5.95 h to 6.22 h it was 1.86 mm. At 5.9 h, there was another rf trip.