DAMPING MECHANISMS
OF TRANSVERSE COLLECTIVE INSTABILITIES FOR SIS 18/100

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Measurements
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high-current oper.,

$\text{Ar}_{40}^{18+}$,

coasting beam,
injection energy

↓

The Slow Wave
with $n = 4$,

$f = 160 \text{ kHz}$,

$\tau = 20 \text{ ms}$
INTRODUCTION

DESIGN VERIFICATION & IMPEDANCE BUDGET

COLLECTIVE INSTABILITY

PREDICT THE STABILITY AND MODE CHARACTERISTICS

DAMPING MECHANISMS
momentum spread ⇒ spread in the betatron frequency

the slip factor $\eta$:

$$\delta\omega_0 = -\eta\omega_0 \frac{\delta p}{p}$$

$+$ due to chromaticity $\xi$:

$$\delta Q = \xi Q \frac{\delta p}{p}$$

\[\downarrow\]

**DISPERSION RELATION FOR LINEAR LANDAU DAMPING:**

$$A \ Z^\perp(\Omega) = - \left[ i\delta\omega_s \int \frac{f(\omega_s)}{\omega_s - \Omega} \, d\omega_s \right]^{-1}$$

with

$$A = \frac{Ne^2}{m\gamma (2\pi)^2 R 2\omega_0 \delta\omega_s} \cdot \frac{\omega_0}{1}$$

⇒ Stability diagram for $(AZ^\perp)$ and $\Omega = \Re e(\Omega) + i\Im m(\Omega)$
coasting beams

at flat-top
 \[ \tau = 4 \text{ ms} \]
\[ (\Delta t \sim 100 \text{ ms}) \]

at injection
 \[ \tau = 3.4 \text{ ms} \]
\[ (\Delta t \sim 15 \text{ ms}) \]

U: Space Charge
   (direct + image charges)

V: Resistive Wall
   (+Kickers+...)

\[ \text{Level contours for } \Im(\Omega) \]
in the plane \[ V + iU = AZ^\perp \]

\[ \Rightarrow \text{other damping mechanisms can become decisive!} \]
NONLINEAR LANDAU DAMPING
(DECOHERENCE)

incoherent tune spread
due to nonlinearities: ● nonlinear space charge
● external nonlinearities (octupoles)
+ complex interplay between them

CONTROVERSIAL ATTITUDES

1. J.L. Laclare, 1985
   H.G. Hereward, 1969
   dispersion relation
   \[\downarrow\]
   nonlinear space-charge effect
   produces damping (stability)

2. D. Möhl, 1974
   dispersion relation
   \[\downarrow\]
   no stability due to nonlinear space-charge
1. dispersion relation
   (Laclare, Hereward)

2. dispersion relation
   (Möhl)

compare

simulations using
the **PATRIC** code

Comparisons: • octupole effect only
  • nonlinear space-charge effect only
  • both effects combined

SIS 18–inspired parameters, coasting beam,
no momentum spread effects, Waterbag distribution
1. Dispersion Relation:

\[ A \, Z^\perp(\Omega) = \left[ i \, \hat{\Delta} \omega_\beta \, \int \frac{d\psi_0}{da} \, \frac{a}{\omega_s(a) - \Omega} \, da \right]^{-1} \]

- Distribution function: \( \psi(a, \varphi) = \psi_0(a) + \psi_1(a, \varphi)e^{-i\Omega t} \)
- \( \hat{\Delta} \omega_\beta \): dsc frequency shift of the equivalent flat-profile beam
- \( \omega_s(a) \) includes tune shifts due to nonlinearities
- \( A Z^\perp \) includes coherent and incoherent impedances (e.g. direct and image charges)
2. Dispersion Relation:

\[
\Delta Q_{coh} \int \frac{d\psi_0}{da} \frac{ada}{\Omega/\omega_0 - (Q_0 + \Delta Q_{inc})} - \\
- \int \frac{d\psi_0}{da} \frac{\Delta Q_{inc} ada}{\Omega/\omega_0 - (Q_0 + \Delta Q_{inc})} = 1
\]

\[
\Delta Q_{coh} \rightarrow AZ_{coh}^\perp
\]

- \(AZ_{coh}^\perp\) includes coherent impedances only (e.g. image charges)
- \(\Delta Q_{inc}\) includes tune shifts due to nonlinear space-charge
- \(Q_0\) includes tune shifts due to external nonlinearities
**Particle Tracking Code** PATRIC

- Particle-In-Cell (PIC) tracking
- sliced approach ("2 $\frac{1}{2} - \text{dim}")
- complete 3D nonlinear particle dynamics
- self-consistent space-charge field
- realistic lattice: transfer matrix (MADX)
- transverse impedance module (Real & Imaginary)
- implementation: C++, developed at HSSP
“particle” distribution in the transverse plane

time evolution for horizontal offset of the beam center
no damping (□)

for comparisons: \( \Re(Z^\perp) = 0 \)
\( \Im(Z^\perp) \) vary

\( \bar{x}(t) \) decays: damping (×)
EXTERNAL NONLINEARITY ONLY

1. dispersion relation

2. dispersion relation

□ → no damping

× → $\bar{x}(t)$ damped
NONLINEAR SPACE-CHARGE EFFECT ONLY

1. dispersion relation

2. dispersion relation

\( U, \text{ (total)} \)

\( U, \text{ (coherent only)} \)

\( V \)

\( U \leftrightarrow \text{no damping} \)

\( \times \leftrightarrow \overline{x}(t) \text{ damped} \)
BOTH NONLINEARITIES COMBINED

1. dispersion relation

2. dispersion relation

□ → no damping
× → $\overline{x}(t)$ damped
SUMMARY

- damping mechanisms of collective instabilities are essential for the Design Verification and Impedance Budget definition

- damping (decoherence) due to nonlinearities can be decisive for GSI synchrotrons SIS 18/100

- substantial (qualitative!) differences between the 1. (Laclare) and the 2. (Möhl) dispersion relation

- simulations using the PATRIC code for nonlinear damping mechanisms and their combination

- comprehensive trilateral comparison supports:
  ★ the 1. dispersion relation
  ★ damping due to the nonlinear space-charge effect